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Updating future reliability of nonlinear systems with low dimensional monitoring data using short-cut simulation

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ABSTRACT

This paper proposes a novel stochastic simulation method of updating future reliability of nonlinear systems with high dimensional uncertainties when the monitoring data is low dimensional. The novelty of the proposed framework is to bypass the most difficult part of the problem: drawing samples of uncertain variables conditioning on the low dimensional monitoring data. This research proposes a short-cut simulation approach: instead of drawing samples of possibly high dimensional uncertain variables conditioning on the monitoring data, it is shown that the problem can be solved by drawing samples of the low dimensional monitoring data conditioning on the future failure event.

1. Introduction

Due to recent demands in infrastructure protection and management, monitoring and assessing the condition of the civil infrastructure have become high-priority research areas. Monitoring data may contain valuable information about the instrumented system and can be used to reduce uncertainties and, in particular, to update reliability of the system due to future loading and excitation.

In the literature [5,8,11,16–18], updating reliability has been discussed under a FORM/SORM framework, where FORM and SORM stand for First-Order and Second-Order Reliability Methods, respectively. Under this framework, monitoring data manifest themselves as "additional events". Given an additional event B, the updated probability of a future failure event F is therefore

$$P(F|B) = \frac{P(F \cap B)}{P(B)} \tag{1}$$

where both the numerator and denominator of (1) can be determined with FORM or SORM. With minimal amount of computation, the FORM/SORM approach works reasonably well for problems with relatively fewer uncertainties. A possible challenge for the FORM/SORM approach is that the dimension of the uncertainties cannot be too high because finding the design point in high dimensional standard Gaussian space is more difficult than in low dimensional space.

In the case that more computation resource is available, a more general reliability updating approach based on Bayesian methods can be employed. To explain the approach, let us divide the uncertain variable Z into two parts $Z = \{\Theta, X\}$: Θ denotes the uncertain system (structural) parameter vector, and X denotes the uncertain excitation. In the case that the future reliability is of concern, the following equation based on the Total Probability Theorem can be used to update future reliability:

$$P(F|d) = \int P(F|\theta)p(\theta|d)d\theta \tag{2}$$

where d denotes the observed monitoring data (D is the monitoring data before it is observed); θ is a specific value of Θ; $P(F|\theta)$ is the future failure probability given the system parameter vector θ; $p(\theta|d)$ is the posterior probability density function (PDF) of Θ. Note that $P(F|\theta)$ can be determined by a reliability analysis where the future excitation is implemented. The Bayesian approach in (2) was taken by Tang [15] to update future reliability for several geotechnical applications. Papadimitriou et al. [13] implement it to update future reliability by observing the posterior PDF asymptotically close to a Gaussian PDF. Estes and Frangopol [6] take visual inspection results of a bridge to first update the PDF of the structural parameters then propagate the uncertainties to obtain the updated reliability.

The evaluation of the integral in (2) is very challenging especially when the θ or X dimension is high. In general, the integral can be estimated through stochastic simulations: if samples of parameter vectors $\{\theta^{(i)}:i = 1,\ldots,n\}$ are drawn from the posterior PDF $p(\theta|d)$, according to the theory of Monte Carlo simulation:

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$$P(F|d) \approx \frac{1}{n}\sum_{i=1}^{n} P(F|\theta^{(i)}) \tag{3}$$

The stochastic simulation approach provides the following benefit: (a) the estimator in (3) is an unbiased estimator of the actual updated failure probability; (b) this approach is applicable for general systems, e.g.: nonlinear dynamical systems, and is applicable even when there are multiple design points.

However, there are, at least, two challenges in the stochastic simulation approach: (a) drawing samples from $p(\theta|d)$ can be highly non-trivial when θ dimension is high; (b) for each sample $\theta^{(i)}$, a reliability analysis is required to determine $P(F|\theta^{(i)})$. This implies that in order to evaluate $P(F|d)$, repetitive reliability analyses are needed. Efforts were taken to resolve the first issue (e.g.: see Beck and Au [2] and Ching and Chen [3]), but so far for problem with Θ dimension more than 20, drawing samples from $p(\theta|d)$ is still very challenging. On the other hand, Cheung and Beck [4] address the latter challenge by introducing an interesting new method based on the following equation:

$$P(F|d) = \frac{p(d|F)P(F)}{p(d|F)P(F) + p(d|F^C)[1 - P(F)]} \tag{4}$$

where F^C is the future non-failure event. They found that the two constants $p(d|F)$ and $p(d|F^C)$ can be readily estimated if samples of $p(\theta|d)$ can be easily drawn, hence the future reliability can be updated without further repetitive reliability analyses. This approach is quite general but still requires drawing samples from $p(\theta|d)$.

What is missing is a technique that can resolve the aforementioned two challenges. In this paper, it is shown that when the monitoring data D is low dimensional, both challenges can be bypassed. In particular, a conjugate approach based on "short-cut simulation" will be developed so that the updating of future reliability no longer requires drawing samples from $p(\theta|d)$; also, the repetitive reliability analyses are not necessary. Therefore, the new approach is applicable even when Θ is extremely high dimensional.

The limitation of the new approach is that it is applicable only when the monitoring data D is low dimensional. Therefore, the goal of this paper is not to propose a method that can outperform previous Bayesian methods in all ways but to propose a method that can be very attractive under a special scenario: when the monitoring data is low dimensional. The limitation of low dimensional data certainly restricts the applications of the proposed method. Nonetheless, for the cases with low dimensional data (e.g.: the two numerical examples in this paper), the proposed method may be superior to other Bayesian methods because it is robust against uncertainty dimension. When the monitoring data is only one-dimensional, an even more efficient algorithm based on subset simulation [1] is developed in the appendix for faster computation.

2. Problem definition

The goal of regular reliability analyses is to estimate future failure probability given the probability distribution of the uncertainties in the target system and the mathematical model M of the system, i.e. to compute $P(F)$. When monitoring data D is available, it is essential to incorporate it to reduce the uncertainties and to update future reliability because D may contain much information about system parameter vector Θ. Therefore, it is desirable to develop a methodology to update future reliability based on these measurements, i.e. to compute $P(F|d)$.

To illustrate the problem of updating future reliability, let us consider the schematics in Fig. 1, where $X^{current}$ denotes the uncertain excitation during the (current) monitoring process, while X^{future} denotes the future uncertain excitation. Note that

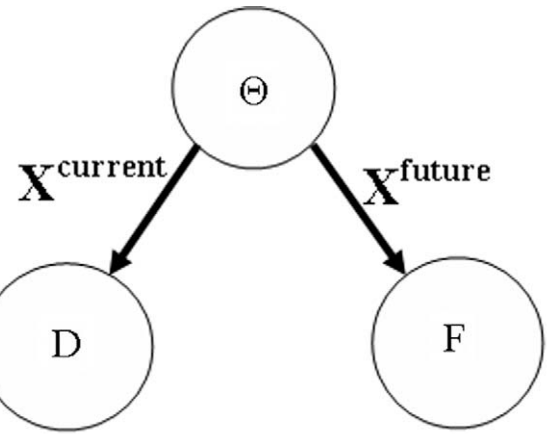

Fig. 1. The graphical model for $X^{current}$, X^{future}, Θ, D and F.

$X^{current}$ and X^{future} can be completely of different type of excitation. It is assumed in this study that the system parameter vector Θ stays constant from the time instant of monitoring to future excitation and that $X^{current}$ and X^{future} are independent.

As an example, let us consider D as the capacity of a pile estimated from a in-situ dynamical pile test, then Θ may contain uncertain soil properties and uncertain parameters in the soil constitutive models, while $X^{current}$ is the uncertain dynamical impact excitation; let F be the failure event under the actual static working load in the future, then X^{future} is the uncertain static working load. It is clear that D depends on Θ and $X^{current}$, while F depends on Θ and X^{future}. For this example, it is reasonable to assume $X^{current}$ (uncertain dynamic impact excitation) and X^{future} (uncertain future static working load) to be independent. This research tries to answer the following question: given the capacity D from the in-situ dynamical test, update the reliability of the pile subject to future static working load.

Please note that under the assumption that $X^{current}$ and X^{future} are independent, D and F are independent conditioning on Θ. That is, if the system parameter vector Θ are known, gaining the information of D will not help to reduce uncertainties in F, and converse is also true. Equivalently,

$$p(d|\theta, F) = p(d|\theta) \quad p(d|\theta, F^C) = p(d|\theta) \tag{5}$$

This property is the key to the derivation of the short-cut simulation approach. In general, it may not be the case that $X^{current}$ and X^{future} are statistically independent. For such cases, the proposed method is not applicable. However, there are cases where this assumption seems quite plausible, especially for the cases where the time between "current" and "future" is reasonably far apart or for the cases where $X^{current}$ and X^{future} are different in nature.

3. Short-cut simulation

The derivations of the short-cut simulation approach start from (4). From (4), it is clear that the estimation of the updated future reliability $P(F|d)$ can be done through the estimation of $P(F)$, $p(d|F)$, and $p(d|F^C)$. Estimating $P(F)$ is the same as an ordinary reliability problem. However, estimating $p(d|F)$ and $p(d|F^C)$, i.e. the PDF of the monitoring data D conditioning on future failure and non-failure events, seems highly non-trivial.

Although estimating $p(d|F)$ and $p(d|F^C)$ is non-trivial, drawing samples from those two PDFs is surprisingly simple as described in the following. Let us start the discussion from the following equations: