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An integrated method for predicting damage and residual tensile strength of composite laminates under low velocity impact

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ABSTRACT

An integrated approach is presented to analyze the whole process of damage initiation and development for composite laminates under impact loading as well as tensile loading after impact using the 3D progressive damage theory. The real impact damage status of composite laminates is employed to analyze the residual tensile strength instead of the artificial premises adopted by traditional methods. This integrated approach can not only improve the prediction accuracy of the ultimate strength but also avoid large numbers of experiments for obtaining the impact damage parameters. A parametric modeling program package based on the analytical method has been developed.

1. Introduction

Fiber reinforced composite laminates are widely used in many engineering fields, especially in aeronautic and aerospace structures owing to their high strength-to-weight and stiffness-to-weight ratios. However, they are susceptible to damage due to low velocity impact loading during manufacture and in service. Low velocity impact loading can cause extensive sub-surface damage that may not be visible on the laminate surface but can lead to a significant reduction in the residual strength of composite laminates. Therefore, considerable research has been done to better understand the impact properties and residual strength after impact.

Kevin and Reza [1] studied the impact damage process of laminates but they did not consider the residual strength after impact (RSAI). Johnson et al. [2] and Allix [3] performed the impact damage analysis in a way similar to that presented in Ref. [1]. Husman et al. [4] proposed a model relating the RSAI of composite laminates to the impact energy, in which the theoretical prediction exhibited good agreement with the experimental results. Caprino and Lopresto [5] presented a model for predicting the RSAI of laminates with an indentation law, where the RSAI was a function of the indentation depth. Although these models exhibit good correlations with the corresponding experimental data, many of the parameters have to be obtained by a number of experiments and the internal damage of laminates cannot be appropriately predicted. To overcome these disadvantages, more detailed stress analyses of the laminates should be performed. Davies et al. [6,7] predicted the threshold impact energy for onset of delamination and the relationship between the impact force and the delamination using the force-driven model and energy-balance model. Good agreement was obtained by comparison the theoretical prediction with the experimental data. More importantly, the CPU time spent on the analysis by the models was greatly shortened. However, Davies et al. did not predict the other failure modes such as transverse matrix cracking, matrix crushing, and fiber failure, which have an insignificant influence on the residual strength of laminates. Furthermore, the prediction error of the threshold impact energy by the models would increase if the fiber breakage takes place. El-Zein and Reifsnider [8] adopted the average stress criterion to predict the RSAI, and the damaged zone due to impact was simulated as an elliptical inclusion. Soutis and Curtis [9] and Gottesman et al. [10] proposed two different models for predicting the RSAI, where the damaged zone generated by impact was based on the assumption about a regular geometry existing in these models. These premises, however, still have two evident drawbacks in that the prediction accuracy of the RSAI cannot be guaranteed and that many parameters have to be obtained by performing numerous experiments.

For the research mentioned above, the impact process is analyzed individually or the RSAI is predicted based on the assumption about impact damage. The objective of this paper is to propose a whole-process analysis approach to exploring the impact process and the RSAI, where the entire damage initiation and development process of composite laminates under the impact loading, the tensile loading after impact and prediction of RSAI are all involved. The 3D progressive damage theory is used in the model. The real

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impact damage status of composite laminates is applied to analyzing the residual tensile strength instead of the artificial premises adopted by many traditional methods for the damage status of composite laminates after impact. This integrated approach can not only improve the prediction accuracy of the ultimate strength but also bypass large numbers of experiments for obtaining the impact damage parameters. Furthermore, the RSAI can be predicted directly from the impact energy by this approach. Based on the analytical method, a parametric modeling program has been developed to predict the impact damage process and RSAI of composite laminates with any ply orientation.

2. Analytical model

The 3D progressive damage theory is applied in the analysis. The theoretical model consists of a stress analysis for determining the stress distributions inside the composite laminates and a failure analysis for predicting the initiation and the extent of the damage. The solving procedure is briefly summarized as follows.

2.1. Stress analysis

2.1.1. Stress analysis for the impact process

For the small strain theory, the equilibrium equation for composite laminates can be expressed as [11]

$$\sigma_{ij,j} = \rho \ddot{u}_i \tag{1}$$

where σ_{ij}, u_i and ρ are stresses, displacements and mass density, respectively.

The stress–strain relation is

$$\sigma_{ij} = C_{ijkl} e_{kl} \tag{2}$$

where C_{ijkl} is the elastic modulus of material, e_{kl} is strain.

By substituting Eq. (1) into Eq. (2), the governing equation of the composite laminates can be written as

$$C_{ijkl} u_{k,lj} = \rho \ddot{u}_i \tag{3}$$

In terms of finite element terminology, the equations of equilibrium for a finite element system in motion can be expressed as

$$\mathbf{M}\underline{\ddot{U}} + \mathbf{K}\underline{U} = F \tag{4}$$

Where **M** and **K** are the mass and stiffness matrices, $\underline{F}$ is the external force vector, $\underline{U}$ and $\underline{\ddot{U}}$ are the displacement and acceleration vectors of the finite element assemblage. The displacements of each element can be calculated by solving Eq. (4) utilizing the initial and boundary conditions. The stresses of each element may then be obtained from Eq. (1).

2.1.2. Stress analysis for the tensile process

For the orthotropic composite laminates, the strain distributions throughout the laminated plate are given by

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y},$$
$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{5}$$

The stress–strain relation is expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & 0 & 0 & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} & 0 & 0 & \bar{Q}_{26} \\ \bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33} & 0 & 0 & \bar{Q}_{36} \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{55} & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{55} & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{36} & 0 & 0 & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} \tag{6}$$

The equilibrium equations can be expressed as

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0, \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0,$$
$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \tag{7}$$

By substituting Eqs. (5) and (6) into Eq. (7), the equilibrium equations can be written as

$$\begin{aligned} &\bar{Q}_{11}\frac{\partial^2 u}{\partial x^2} + \bar{Q}_{66}\frac{\partial^2 u}{\partial y^2} + \bar{Q}_{55}\frac{\partial^2 u}{\partial z^2} + 2\bar{Q}_{16}\frac{\partial^2 u}{\partial x \partial y} + \bar{Q}_{16}\frac{\partial^2 v}{\partial x^2} + \bar{Q}_{26}\frac{\partial^2 v}{\partial y^2} \\ &\quad + \bar{Q}_{45}\frac{\partial^2 v}{\partial z^2} + (\bar{Q}_{12} + \bar{Q}_{66})\frac{\partial^2 v}{\partial x \partial y} + (\bar{Q}_{13} + \bar{Q}_{55})\frac{\partial^2 w}{\partial z \partial x} \\ &\quad + (\bar{Q}_{36} + \bar{Q}_{45})\frac{\partial^2 w}{\partial y \partial z} = 0 \end{aligned} \tag{8}$$

$$\begin{aligned} &\bar{Q}_{16}\frac{\partial^2 u}{\partial x^2} + \bar{Q}_{26}\frac{\partial^2 u}{\partial y^2} + \bar{Q}_{45}\frac{\partial^2 u}{\partial z^2} + (\bar{Q}_{12} + \bar{Q}_{66})\frac{\partial^2 u}{\partial x \partial y} + \bar{Q}_{66}\frac{\partial^2 v}{\partial x^2} \\ &\quad + \bar{Q}_{22}\frac{\partial^2 v}{\partial y^2} + \bar{Q}_{44}\frac{\partial^2 v}{\partial z^2} + 2\bar{Q}_{26}\frac{\partial^2 v}{\partial x \partial y} + (\bar{Q}_{36} + \bar{Q}_{45})\frac{\partial^2 w}{\partial z \partial x} \\ &\quad + (\bar{Q}_{23} + \bar{Q}_{44})\frac{\partial^2 w}{\partial y \partial z} = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} &(\bar{Q}_{36} + \bar{Q}_{45})\frac{\partial^2 u}{\partial y \partial z} + (\bar{Q}_{13} + \bar{Q}_{55})\frac{\partial^2 u}{\partial z \partial x} + (\bar{Q}_{44} + \bar{Q}_{23})\frac{\partial^2 v}{\partial y \partial z} \\ &\quad + (\bar{Q}_{36} + \bar{Q}_{45})\frac{\partial^2 v}{\partial z \partial x} + \bar{Q}_{55}\frac{\partial^2 w}{\partial x^2} + \bar{Q}_{44}\frac{\partial^2 w}{\partial y^2} + \bar{Q}_{33}\frac{\partial^2 w}{\partial z^2} \\ &\quad + 2\bar{Q}_{45}\frac{\partial^2 w}{\partial y \partial z} = 0 \end{aligned} \tag{10}$$

where the $\bar{Q}_{ij}$ is related to the material constants and the ply orientation of the layer in a laminate. From Eqs. (8)–(10), the displacements of each element can be calculated by the finite element method utilizing the initial and boundary conditions. The strain and stress of each element can then be obtained from Eqs. (5) and (6), respectively.

2.2. Failure analysis

2.2.1. Failure criteria for the impact process

Typical low velocity impact damage appears in the form of transverse matrix cracking, matrix crushing, delamination and fiber fracture. Hou et al. [12] summarized the stress-based failure criteria for the four types of damage. These criteria have been adopted in many previous researches [13,14]. Here, we adopt the criteria presented in Ref. [12], which are formulated below:

Transverse matrix cracking

$$e_m^2 = \left(\frac{\sigma_{22}}{Y_t}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 + \left(\frac{\sigma_{23}}{S_{m23}}\right)^2 \geqslant 1 \quad (\sigma_{22} \geqslant 0) \tag{11}$$

Table 1
Material property degradation rules.

Damage mode	Property degradation rule
Matrix cracking	$E_{yy}^* = 0.2E_{yy}$ $G_{xy}^* = 0.2G_{xy}$ $G_{yz}^* = 0.2G_{yz}$
Matrix crushing	$E_{yy}^* = 0.4E_{yy}$ $G_{xy}^* = 0.4G_{xy}$ $G_{yz}^* = 0.4G_{yz}$
Fiber–matrix shear-out	$G_{xy}^* = \upsilon_{xy}^* = 0$
Delamination	$E_{zz}^* = G_{yz}^* = G_{xz}^* = \upsilon_{yz}^* = \upsilon_{xz}^* = 0$
Fiber failure	$E_{xx}^* = 0.07E_{xx}$

Note: Superscript "*" indicates the material property after degradation.