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A dynamic stiffness element for free vibration analysis of composite beams and its application to aircraft wings

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Abstract

The dynamic stiffness matrix of a composite beam that exhibits both geometric and material coupling between bending and torsional motions is developed and subsequently used to investigate its free vibration characteristics. The formulation is based on Hamilton's principle leading to the governing differential equations of motion in free vibration, which are solved in closed analytical form for harmonic oscillation. By applying the boundary conditions the frequency dependent dynamic stiffness matrix that relates the amplitudes of loads to those of responses is then derived. Finally the Wittrick–Williams algorithm is applied to the resulting dynamic stiffness matrix to compute the natural frequencies and mode shapes of an illustrative example. The results are discussed and some conclusions are drawn. The theory can be applied for modal analysis of high aspect ratio composite wings and can be further extended to aeroelastic studies.
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Keywords: Dynamic stiffness method; Free vibration; Composite beams; Geometrical and material coupling; Wittrick–Williams algorithm

1. Introduction

The free vibration analysis of composite beams [1–7] is an important and well-known area of research, particularly because of its practical applications in aeronautical design [8,9]. Such an analysis is generally considered as a prerequisite to carry out aeroelastic or response analysis [10–14]. The literature on the free vibration analysis of composite beams is dominated by finite element and other approximate methods [4,5] although some results using the dynamic stiffness method are also available [15]. It is well recognised that the free vibration and response behaviour of composite beams can be very different from their metallic counter parts. This is primarily due to coupling between various modes of deformation that can occur in fibrous composites as a result of their anisotropic properties, but cannot generally happen in isotropic metals. Changing the ply orientation of the fibres can control the coupling to achieve desirable dynamic effects, which is not possible with conventional materials such as aluminium and steel. From an aeroelastic point of view, especially when designing composite wings this is significant. Furthermore, the particular type of coupling between the bending and torsional motion in a high aspect ratio aircraft wing can cause instability such as flutter, which is of considerable interest [9–11,13,14] in aeronautical design.

The current paper is concerned with the dynamic stiffness formulation and free vibration analysis of composite beams that exhibit bending–torsion coupling. In the analysis the dynamic stiffness method is preferred because unlike the finite element or other approximate methods, it gives exact results obtained from exact mass and stiffness representation of the structure. Essentially, the method relies on one frequency dependent matrix, called the dynamic stiffness matrix, which has mass and stiffness properties of the structure together, rather than two separate mass and

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stiffness matrices, respectively. The dynamic stiffness matrix of an individual element in a structure is derived from the governing differential equation of motion of the element in free vibration so that all assumptions being within the limits of the differential equation are not severe. The assembly procedure of individual element matrices to form the overall dynamic stiffness matrix of the final structure is similar to that of the finite element method, but the solution technique is different because the formulation in the dynamic stiffness method leads to a non-linear eigenvalue problem instead of the usual linear one used in the finite element method. The Wittrick–Williams algorithm [16] is generally used as solution technique in the dynamic stiffness method yielding all required eigenvalues of the system which are natural frequencies in free vibration and critical buckling loads in elastic buckling problems.

The type of bending–torsion coupling for composite beams considered in this paper arises from two principal sources. One of the sources stems from non-coincident shear centre and centroid of the beam cross-section (This is a common feature in metallic or composite beams of asymmetric cross-section of which an aircraft wing [17] is a classic example). This type of coupling is referred to as geometric coupling as it involves only the geometry of the cross-section. The nature of this coupling is inertial and as a consequence, the bending and torsional motions are uncoupled under static loads. The other type of coupling considered here is applicable to composite beams because it occurs due to anisotropic material properties (caused by fibre orientations). Clearly this latter type of coupling is possible under both static and dynamic loads. This particular form of coupling is referred to as material coupling because it is dependent on the material properties only. Even for a doubly symmetric cross-section (for example, a solid rectangular or a closed box section), bending–torsion coupling can still occur in a composite beam due to ply orientations in the laminate [1,2,6,15]. As this form of coupling is due entirely to material properties, the geometry of the cross-section is not directly relevant.

It is clear from the above discussion that if the cross-section of a laminated composite beam is such that the shear centre and mass centre are non-coincident, both geometric and material coupling can occur and they both need to be considered when studying the free vibration characteristics. In this context a high aspect ratio composite wing is a potential candidate for research that partly motivated this work. It is also important to note that by splitting the coupling terms into geometric and material components offers a better understanding of the problem. In particular, the individual or joint contributions of the coupling induced by each of the two sources to the resulting free vibratory motion of the beam or wing provide considerable insight into the problem.

However, it should be recognised that the bending–torsion coupling effect arising from independent consideration of geometrical and material properties has previously been studied for free vibration analysis of metallic [17] and composite [15] beams, respectively. It is therefore, pertinent and timely in the present study to develop the dynamic stiffness matrix of a bending–torsion coupled composite beam by taking into account the effects of both the geometric and material coupling in a unitary manner in order to investigate the free vibration characteristics. The governing differential equations of motion of the composite beam are derived using Hamilton's principle and the equations are solved analytically. The solution is used to develop the dynamic stiffness matrix, which relates harmonically varying forces with harmonically varying displacements at the ends of the composite beam. Finally the resulting dynamic stiffness matrix is used in conjunction with the Wittrick–Williams algorithm [16] to compute the natural frequencies and mode shapes of an illustrative example. The results are discussed and some conclusions are drawn.

2. Theory

2.1. Derivation of the governing differential equations

A composite beam that exhibits both geometric and material coupling such as an aircraft wing is shown in Fig. 1 in a right-handed co-ordinate system. The elastic axis, which coincides with the Y-axis, is chosen to be the locus of the geometric shear centres of the wing cross-section. It is allowed to deflect out of the plane by $h(y, t)$, whilst the cross-section is allowed to rotate about OY by $\psi(y, t)$, where y and t denote distance from the origin and time, respectively. The wing has a length of L, bending rigidity EI, torsional rigidity GJ, bending–torsion coupling rigidity K, mass per unit length m, and mass moment of inertia per unit length I_α about the Y-axis, respectively. In the figure, x_α is the distance between the mass and elastic axes, which are, respectively, the loci of the mass centre (centroid) and the shear centre of the wing cross-sections, and is positive when the mass axis is aft of the elastic axis as shown. The two principal parameters that are responsible for the geometric and material coupling are x_α and K, respectively. It should be noted that the theory developed does not include the effects of shear deformation and rotatory inertia. It is thus suitable for composite beams for which the cross-sectional dimensions are small compared to the length, for example, a high aspect ratio aircraft wing. The effects of shear deformation and rotatory inertia can be

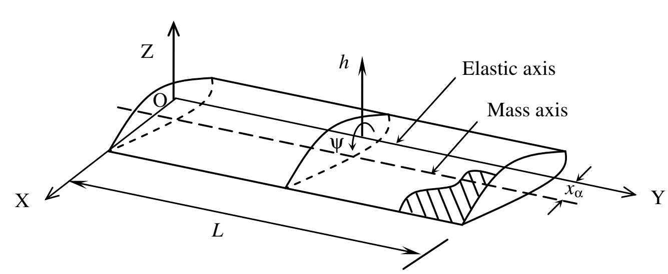

Fig. 1. The co-ordinate system and notation for a bending–torsion coupled composite beam.

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