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A generalised technique for fracture analysis of cracked plates under combined tensile, bending and shear loads

G.S. Palani [a,*], Nagesh R. Iyer [a], B. Dattaguru [b]

[a] *Structural Engineering Research Centre, CSIR Campus, Taramani, Chennai 600 113, India*
[b] *Indian Institute of Science, Bangalore, India*

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Abstract

The objective of this paper is to propose a generalized technique called numerically integrated modified virtual crack closure integral (NI-MVCCI) technique for fracture analysis of cracked plates under combined tensile, bending and shear loads. NI-MVCCI technique is used for post-processing the results of finite element analysis (FEA) for computation of strain energy release rate (SERR) components and the corresponding stress intensity factor (SIF) for cracked plates. NI-MVCCI technique has been demonstrated for 4-noded, 8-noded (regular and quarter-point) and 9-noded (regular and quarter-point) isoparametric plate finite elements. These elements are based on Mindlin's plate theory that considers shear deformation. For all the elements, reduced integration/selective reduced integration techniques have been employed in the studies. In addition, for 9-noded element assumed shear interpolation functions have been used to overcome the shear locking problem. Numerical studies on fracture analysis of plates subjected to tension–moment and tension–shear loads have been conducted employing these elements. It is observed that among these elements, the 9-noded Lagrangian plate element with assumed shear interpolation functions exhibits better performance for fracture analysis of cracked plates.

Keywords: Fracture mechanics; Finite element method; Plates; Tensile; Bending and shear loads; Stress intensity factor; Strain energy release rate; Numerical integration

1. Introduction

Plate/shell formulations are widely used to analyze thin-walled structures such as aircraft fuselages, ship hull components and offshore deck platforms subjected to bending and pressure loads. Through-thickness cracks generally develop when these structures are subjected to cyclic loads and the determination of SIF is critical to the evaluation of fatigue crack propagation. When these plate/shell panels containing through-thickness cracks are subjected to tension, moment and shear loadings, the important crack parameters, SIF, moment intensity factors and shear force intensity factors should be determined accurately for fracture analysis. Problems in fracture of plates subjected to combined tensile, bending and shear loads have been the focus of research for more than three decades [1]. Although many studies have been conducted on fracture of plates, there is still lack of general theory [2] that can be applied to arbitrary geometry and loading. There is scope for development of robust numerical methods to evaluate the fracture parameters and to simulate crack growth in thin plates under bending loads. For the cracked plates subjected to tensile loads, many effective FEA software [3–5] were developed and some of them are available for engineers. On the other hand, for the cracked plates under bending loads, only a few numerical solutions using the finite element method (FEM) are available.

Sih [1] presented procedures to evaluate SIF at the crack tip for plates subjected to tensile and bending loads using the fourth order classical plate theory. The Kirchhoff

* Corresponding author. Tel.: +91 44 22549207; fax: +91 44 22441508.
E-mail address: pal@sercm.csir.res.in (G.S. Palani).

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theory was employed such that the three physically distinct boundary conditions on the crack surface were reduced to two approximate boundary conditions. To improve the accuracy, sixth order Reissner–Mindlin plate theory was proposed by Knowles and Wang [6] to conduct fracture analysis of elastic cracked plates under symmetrical bending loads. Sih et al. [7] provided the expressions for evaluating SIF associated with uniform bending moment, twisting moment and transverse shear force for an infinite plate with a through-the-thickness central crack. Hartranft and Sih [8] generalized this solution to include the effect of the plate thickness on the bending stress distribution. They concluded that the stresses near the crack tip tend to infinity and are proportional to the square root of the distance from the crack tip, the same behaviour as in-plane loading.

The problem of fracture analysis of cracked plates under bending loads was also extensively studied by using FEM. Barsoum [9] proposed a quarter-point thin or thick plate/shell element derived from a degenerate 20-node 3-D isoparametric element for fracture analysis of cracked plates under bending loads and found that his results compare well with those obtained by Hartranft and Sih [8]. The singularity of strains was achieved by placing the mid-side nodes near the crack tip at the quarter-points. Rhee and Atluri [10] developed a special crack tip element for plate bending problems based on the stress hybrid FE technique. In this method, an asymptotic singular stress field satisfying the plate equilibrium equation and singular boundary displacement field is embedded in the crack tip element. They also investigated the effect of an explicit enforcement of the traction-free conditions along the crack surface through variational principle and obtained an accurate solution.

A special crack tip element, called ‘hybrid mongrel singular element’ was proposed by Kang and De Saxce [11]. One of the advantages of this element is that the previously mentioned cumbersome work in the hybrid approach is avoided and monotonic convergence is ensured. This is found to satisfy the stress singularity by using the theoretical asymptotic solutions as an assumed stress hybrid singular element and the strain singularity by manipulating the mid-side node at the quarter-point position similar to Barsoum’s displacement based singular element [9]. Xiao et al. [12] proposed two special hybrid stress elements based on a penalty Hellinger–Reissner principle for analyzing crack problems. Path independent J_1- and I_1^*-integrals are employed for the determination of the stress intensity factor (SIF) at the tip of a crack in mode I or II in a linear isotropic or orthotropic material. Young and Sun [13,14] presented methodologies based on both classical and Reissner–Mindlin theories for computing G for a through-thickness crack in an infinite plate subjected to out-of-plane uniform bending moment and out-of-plane tearing loads using the virtual crack extension and the variation of potential energy. It is shown that SERR obtained using Reissner–Mindlin theory approaches the classical plate solution as the ratio of plate thickness to crack size becomes infinitesimally small. From this energy consideration, the limiting expression of the SIF associated with Reissner–Mindlin theory was explicitly obtained. Hui and Zehnder [15] derived a universal relationship between Kirchoff theory SIF and Reissner–Mindllin theory SIF for thin plates using J-integral technique. They found that in the thin plate limit, the mode I moment intensity factor based on Reissner–Mindlin theory would converge to $\sqrt{(1+\nu)/(3+\nu)}$ (ν = Poisson’s ratio). Viz et al. [16] proposed the procedure for computing the membrane and bending SIF for thin cracked plates using virtual crack extension, nodal release and modified crack closure integral (MCCI) methods. Kirchoff plate theory and plane stress elasticity is used for computing SIF due to bending loads and membrane loads respectively.

The finite elements based on Reissner–Mindlin theory places fewer restrictions on the smoothness of the approximating space, so that the plate displacements can be approximated with C^0 interpolations [17]. Despite these advantages, FE formulations based on Reissner–Mindlin theory for fracture analysis are complicated by shear-locking and the calculation of mixed-mode SIF. Analogous to SIF of classical 3-D fracture mechanics, in Reissner–Mindlin theory the near-tip fields are described in terms of moment and shear force intensity factors. The definition of these quantities is presented by Sih [1] with analytical solutions for some simple geometry. More recent analytical and numerical results are also limited to simple geometries and quite often are only available for pure mode-I loading. When considering FE approximations of relatively thin plates, it is necessary to address the phenomenon known as shear locking. As the plate becomes very thin, both the solution and its approximation must satisfy the constraint that the transverse shear strains vanish in the domain. When there are not enough functions in FE subspace(s) which satisfy this constraint, a poor approximation results for the plate displacements. Quite a few plate finite elements are available in the literature [18,19], which do not exhibit shear locking. The free surface of the crack faces present additional challenges and some elements perform better than others. In view these, there exist scope for development of procedures for fracture analysis of plates subjected to combined tensile, bending and shear loads by using linear elastic fracture mechanics (LEFM) principles. Number of finite elements based on Reissner–Mindlin theory were proposed in the literature for plate bending problems, which accounts for shear deformations. Among these plate finite elements, the isoparametric family of finite elements are generally employed for FEA of plates. The preferred ones among these are the 4-noded bilinear element (QUAD4), 8-noded Serendipity element and the 9-noded Lagrangian element. Performance of these elements varies depending on the choice of the numerical integration order for the stiffness computation or the formulation used for representation of the shear energy component of the stiffness matrix. These aspects have been very well studied [18,19] for the performance on static analysis of plates.