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A multi-criteria leave-two-out cross-validation procedure for max-stable process selection

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ABSTRACT

Max-stable processes are the extension of the univariate extreme value theory to the spatial case. Contrary to the univariate case, there is no unique parametric form for the limiting distribution in the spatial case, and several max-stable processes can be found in the literature. Selecting the best of them for the data under study is still an open question. This paper proposes a procedure for discriminating max-stable processes by focusing on their spatial dependence structure. Specifically, it combines a leave-two-out cross-validation scheme and a large panel of adapted criteria. We compare five of the most commonly used max-stable processes, using as a case study a large data set of winter maxima of 3-day precipitation amounts in the French Alps (90 stations from 1958 to 2012). All the introduced criteria show that the extremal-t, geometric Gaussian and Brown–Resnick processes are equally able to represent the structure of dependence of the data, regardless of the number of stations or years. Although these results have to be confirmed by replicating the study in other contexts, they may be valid for a wide range of environmental applications.

1. Introduction

Model selection is a classical issue in statistics, whether to make a choice between several families of parametric models or to make a selection between several explanatory variables. A proper

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model selection should be done through the use of statistical criteria, which should be able to measure the ability of a model to predict or explain data, or to compare and rank models. The well-known coefficient of determination R^2 (Barrett, 1974) and the adjusted coefficient of determination $\hat{R}^2$ (Srivastava et al., 1995) consider the proportion of total variability explained by the model. Likelihood-ratio tests (Vuong, 1989) can be used to compare nested models. Non-nested models can be compared using Bayes factor (Kass and Raftery, 1995) or likelihood based criteria such as the Akaike Information Criterion (AIC, Akaike, 1974) or the Bayesian Information Criterion (BIC, Schwarz et al., 1978). However, although these criteria are able to compare models, they do not measure the suitability of a given model for prediction. A classical method for assessing the predictive efficiency of a model is cross-validation (Arlot et al., 2010) which consists in splitting data, once or several times, into two parts: the first one is used to fit the model and the second one for validation (Pujol et al., 2007; Blanchet and Lehning, 2010; Westra et al., 2013).

In extreme value statistics (Coles, 2001), one generally extrapolates far beyond the highest recorded observation, mostly for risk mitigation purpose. That is why the predictive ability of the models is particularly crucial. A proper model selection in extreme value statistics should therefore particularly consider the predictive qualities of the models in competition. In the "block maxima" approach of univariate extreme value theory, the Fisher–Tippett–Gnedenko theorem (Fisher and Tippett, 1928; Gnedenko, 1943) ensures that the limiting distribution for maxima of random variables is the GEV (Generalized Extreme Value) distribution. Therefore, in this case, model selection does not rely on the choice of the distribution, but rather on the choice of covariates for GEV parameters or, ultimately, on the sign of the shape parameter which determines the type of distribution (Fréchet, Gumbel or reversed Weibull). However, in the multivariate case, there is no unique parametric form for the limiting distribution and it is necessary to find a parametric extreme value copula flexible enough to capture the structure of dependence (Ribatet and Sedki, 2012; Davison et al., 2012), or to work in a nonparametric framework (Capéraà et al., 1997; Zhang et al., 2008). Max-stable processes (de Haan, 1984) are the extension of the multivariate GEV distribution to the infinite dimension and are mostly used in a spatial context (Davison et al., 2012; Ribatet, 2013). Max-stable processes are usually considered with unit Fréchet margins, and in this case they all may be expressed through the de Haan's spectral representation (de Haan, 1984). However, several parametric distributions resulting from this representation exist, each proposal of the literature representing a specific way of modeling the spatial dependence structure of extreme values. Model selection is thus an important issue.

The first two max-stable processes to be introduced are the Smith (1990) and Schlather (2002) processes. However, they have some major drawbacks. The Smith process provides too smooth realizations which are usually not realistic (Reich and Shaby, 2012; Wadsworth and Tawn, 2012). The Schlather process assumes rather strong dependence in extremes (extremal dependence) at two locations regardless of the distance apart, which is questionable for many data (Blanchet and Davison, 2011; Davison et al., 2012). One way to solve this difficulty is to divide the studied area into several sub-regions in which the non-independence assumption of the Schlather process may hold (Blanchet and Davison, 2011; Lee et al., 2013). However, the choice of these sub-regions may be an issue in itself. Several new max-stable processes were introduced recently to solve these drawbacks. Smith and Stephenson (2009) suggested an extended Smith process. The Brown–Resnick (Kabluchko et al., 2009) and geometric Gaussian (Davison et al., 2012) processes have similar expression of the joint distribution as the Smith process but with more realistic realizations. Reich and Shaby (2012) proposed a max-stable process connected to the Smith process but including a nugget term and thus producing also more realistic realizations. Wadsworth and Tawn (2012) introduced the Gaussian–Gaussian process which is a superposition of the Smith and Schlather processes in order to keep the advantages of both families without their drawbacks. Davison and Gholamrezaee (2012) suggested to use a truncated Schlather process instead of the classical Schlather process with the aim of reaching independence in extremes far apart. The extremal-t process (Opitz, 2013) is a generalization of the Schlather process with an additional parameter controlling the extremal dependence between locations far apart. Recently, the geometric Gaussian process was extended to the even more flexible Tukey process (Xu and Genton, 2016).

Several applications of max-stable processes (Shang et al., 2011; Westra and Sisson, 2011; Lee et al., 2013; Raillard et al., 2014; Zhang et al., 2014) make the choice of considering only one