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On the link between natural emergence and manifestation of a fundamental non-Gaussian geostatistical property: Asymmetry

Philipp Guthke *, András Bárdossy

Institute for Modelling Hydraulic and Environmental Systems, Dept. of Hydrology and Geohydrology, University of Stuttgart, Pfaffenwaldring 61, D-70569 Stuttgart, Germany

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ABSTRACT

The geostatistical workflow of data analysis, model fitting, and subsequent interpolation or simulation has recently been enhanced by several methods. These methods can be summarized under the terms 'rank-order geostatistics', for the empirical analysis part of the workflow, and 'copula geostatistics', for the theoretical foundation and the modeling (interpolation or simulation). Besides taking into account non-Gaussianity, the main advantage of this alternative way to treat geostatistical problems is the descriptive power and standardized interpretability. This paper addresses the empirical analysis part of the workflow. We investigate the interplay between structural features, statistical properties, and underlying dynamic processes of a realization of a spatial field.

(1) In the first part of this paper we recapitulate and consolidate the advances in the empirical analysis part of the geostatistical workflow and put them into context of 'classical' geostatistics. We place particular emphasis on the theoretical foundation of so called asymmetry functions, because in our opinion they are the necessary first step away from Gaussian geostatistics. (2) In the second part, we rigorously analyze how specific types of structural features are related to the asymmetry of a spatial field. In the geostatistical tradition of interpreting the shape of variograms or correlation functions, we give examples of how to interpret different shapes of the asymmetry function. (3) We subsequently report how dynamic processes naturally lead to the emergence of asymmetrical (non-Gaussian) spatial structures. To demonstrate

* Correspondence to: TransnetBW, Pariser Platz, Osloer Straße 15-17, 70173 Stuttgart, Germany.
E-mail address: p.guthke@web.de (P. Guthke).

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these findings, we investigate the manifestation of different spatial data sets (land surface elevation, groundwater contamination, grades of an undergraduate exam) and relate the non-Gaussian structural features to the underlying dynamic processes. Numerical process models are utilized to manifest evidence of how realistic processes naturally lead to complex non-Gaussian structures.

The key purpose of this paper is to show that asymmetry is a fundamental geostatistical property and a result of different kinds of spatial processes.

1. Introduction

Naturally occurring spatially distributed variables (the manifestation of some characteristic in space) usually exhibit a considerable amount of heterogeneity and complex structural features. It has already been shown in many studies that heterogeneous spatial fields cannot be modeled appropriately by means of 'classical' geostatistics (with the assumption of multivariate Gaussianity): Bárdossy and Li (2008) show that groundwater quality parameters (concentrations of certain solutes) can be interpolated more realistically with a non-Gaussian copula as the spatial random function, Haslauer et al. (2012) demonstrate that the hydraulic conductivity data set of Borden exhibits non-Gaussian bivariate copulas, Rossi et al. (1992) study spatial patterns of organisms that cluster in a non-Gaussian manner, and Bárdossy and Pegram (2012) find evidence that rain over certain areas might have non-Gaussian higher order interdependencies.

It has further been studied and is documented well in the literature, how *'models go bad'* (Zinn, 2003) and lead to wrong conclusions when using multivariate Gaussian fields as input for subsequent (numerical) models or risk assessment: Among others, Journel and Alabert (1989), Gómez-Hernández and Wen (1998), Zinn (2003), and Haslauer (2011) all support, with different studies, the importance of a realistic (non-Gaussian) description of structural features.

Given the importance of realistic spatial modeling, in this paper we add an additional aspect to the workflow of spatial analysis. Although spatial data sets are often available only as a single snapshot in time, they (and their structural features) certainly emerge as a result of several underlying dynamic physical processes. Instead of solely looking at the statistics of an existing realization of a spatial field, we also investigate the processes that lead to a given manifestation of a spatially distributed variable, and relate those processes to non-Gaussian statistics.

We further demonstrate that different kinds of dynamic processes lead to the same fundamental manifestation of non-Gaussianity: asymmetry.

As an introductory example of what motivates this work, consider land surface elevation as the spatially distributed variable. Geological, biological, or erosive processes lead to the formation of mountains and valleys of different shapes, forms, and roughnesses, depending on the erosive regime in the area under study. In water-dominated regions there is at least one fundamental difference between mountains and valleys. That is that mountains (the high values) are always surrounded by closed isolines, while valleys (the low values) build interconnected (river-) structures that always descend downwards towards the boundary of the investigated area. Loosely topologically speaking, there is a high chance that the minimum value is found at the boundary of the domain. In arid regions, as a counterexample, low values might form local minimums because there is no forcing (water flow) leading to interconnected low value channels.

The essence of our point is that geostatistical investigations should not only aim to describe structural features with suitable statistics, they should also consider the underlying dynamic processes in order to complete the whole picture of a spatially distributed variable. Not only the question 'how' but also the question 'why is something structured as it is' should be answered in