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Modelling spatial heteroskedasticity by volatility modulated moving averages

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ABSTRACT

Spatial heteroskedasticity has been observed in many spatial data applications such as air pollution and vegetation. We propose a model, the volatility modulated moving average, to account for changing variances across space. This stochastic process is driven by Gaussian noise and involves a stochastic volatility field. It is conditionally non-stationary but unconditionally stationary: a useful property for theory and practice. We develop a discrete convolution algorithm as well as a two-step moments-matching estimation method for simulation and inference respectively. These are tested via simulation experiments and the consistency of the estimators is proved under suitable double asymptotics. To illustrate the advantages that this model has over the usual Gaussian moving average or process convolution, sea surface temperature anomaly data from the International Research Institute for Climate and Society are analysed.

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1. Introduction

A classical assumption made when dealing with spatial data is that the variance is a constant and the covariance between measurements at two locations is a function of their distance apart. In practice, however, it has been observed that this does not hold for many data sets and accounting for spatial heteroskedasticity or spatial volatility has multiple benefits.

The first benefit is the better representation of the data. In a recent paper, it was shown that including spatial volatility in road topography models better captures the hilliness features of the

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roads (Johannesson et al., 2016). This has implications on estimating the risk of vehicle damage and simulating fuel consumption. In some settings, the presence of spatial volatility can be also explained. For example, in a study of sulphur dioxide concentrations by Fuentes and Smith (2001), it was found that states which lie close to several coal power plants tend to have high variability in their readings. This was attributed to the dependence of the levels on the wind speed, the wind direction, as well as the atmospheric stability.

A second benefit of modelling spatial volatility is the potential for improving prediction. This was seen by Huang et al. (2011) when they fitted a Gaussian process with volatility to vegetation and nitrate deposition data. In the case of agriculture yields, prediction intervals accounting for spatial volatility will be useful for insurance companies when they set crop insurance prices (Yan, 2007).

Another way of using spatial volatility would be as an indicator of regime change. Such an approach has been taken in desertification and urban planning studies (Seekell and Dakos, 2015; Getis, 2015). In the first case, regions of high volatility demarcate the bare and the extensive vegetative cover; while in the second case, it is used to identify slum areas.

In this paper, we introduce stochastic volatility to the well-known Gaussian moving average (GMA) or process convolution model:

$$Y(\mathbf{x}) = \int_{\mathbb{R}^d} g(\mathbf{x}-\boldsymbol{\xi})W(\mathrm{d}\boldsymbol{\xi}), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^d$ for some $d \in \mathbb{N}$, g is a deterministic (kernel) function and W is the homogeneous standard Gaussian basis on $\mathbb{R}^d$ whose Lévy seed (which we shall define in Section 2) has mean 0 and variance 1. This results in the so-called volatility modulated moving average (VMMA):

$$Y(\mathbf{x}) = \int_{\mathbb{R}^d} g(\mathbf{x}-\boldsymbol{\xi})\sigma(\boldsymbol{\xi})W(\mathrm{d}\boldsymbol{\xi}), \tag{2}$$

where $\{\sigma^2(\boldsymbol{\xi}) : \boldsymbol{\xi} \in \mathbb{R}^d\}$ is a stationary stochastic volatility field, independent of W. In Huang et al. (2011), the stochastic volatility is multiplied as a factor to the main spatial process; here, it appears as an integrand. As such, Y can sometimes be identified as a solution to a stochastic partial differential equation (SPDE). Following similar arguments to those on page 559 of Bolin (2014), we find that Y can be viewed as a solution to:

$$(\kappa^2 - \Delta)^{\alpha/2} Y(\mathbf{x}) = \sigma(\mathbf{x})\dot{W}(\mathbf{x}),$$

where $\alpha > d/2$, $\kappa > 0$, $\Delta = \sum_{i=1}^{d} \partial^2/\partial x_i^2$ is the Laplacian operator and $\dot{W}$ is Gaussian white noise, when g is a Matérn kernel defined by:

$$g(\mathbf{x}-\boldsymbol{\xi}) = 2^{1-(\alpha-d)/2}(\kappa|\mathbf{x}-\boldsymbol{\xi}|)^{(\alpha-d)/2} K_{(\alpha-d)/2}(\kappa|\mathbf{x}-\boldsymbol{\xi}|)/[(4\pi)^{d/2}\Gamma(\alpha/2)\kappa^{\alpha-d}], \tag{3}$$

and $K_{(\alpha-d)/2}$ is the modified Bessel function of the second kind.

VMMAs can be seen as an extension of the type G Lévy moving average (LMA) recently studied by Bolin (2014) and Wallin and Bolin (2015):

$$Y(\mathbf{x}) = \int_{\mathbb{R}^d} g(\mathbf{x}-\boldsymbol{\xi})L(\mathrm{d}\boldsymbol{\xi}), \tag{4}$$

where L is a (homogeneous) type G Lévy basis. This means that the Lévy seed, $L' \stackrel{d}{=} V^{1/2}Z$ where V is an infinitely divisible random variable and Z is a standard normal random variable independent of V. This is equivalent to restricting σ^2 in the definition of our VMMA to be infinitely divisible and independent across locations, which would be a rather strong assumption to make.

In Bolin (2014), an expectation–maximisation (EM) algorithm is used to conduct inference for a Laplace noise-driven LMA with Matérn covariance. This relies on the connection of the spatial field to an SPDE and Hilbert space approximations via basis functions. The resulting sparse covariance matrices help to make the procedure computationally efficient. In Wallin and Bolin (2015), this is extended to a Monte Carlo EM algorithm to handle cases where there is an additional trend involving covariates and the E-step cannot be calculated analytically. These EM algorithms make use of the

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