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Half-spectral space–time covariance models

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ABSTRACT

We develop a new class of space–time Gaussian process models by specifying covariance functions using what we call a half-spectral representation. We establish general properties of half-spectral models and appeal to results on a screening effect analysis to claim more natural space–time interaction properties in the class of models we develop. To test this claim and to more generally test this class of models, we fit models we develop in this paper to a wind power dataset. We show our models fit these data better than other common separable and non-separable space–time models.

1. Introduction

Continuous natural phenomena in space or space–time are often modeled statistically as Gaussian processes. Generally, a Gaussian process model requires specification of a mean structure and a covariance or dependence structure. Since covariance models must be positive definite, mathematically valid dependence structures can be quite difficult to define. Therefore, a primary focus in defining new Gaussian process models is on defining valid dependence structures. In this paper, we develop properties of a specific set of stationary space–time dependence structures using what we call a half-spectral approach. We present a new class of models built using this half-spectral approach and further develop properties of this class. In particular, our theoretical exploration of these models focuses on space–time interaction properties, and we show by way of an application that careful consideration of space–time interaction can lead to better fitting models.

Half-spectral approaches to defining dependence structures have been studied previously by several authors (Stein, 2005; Cressie and Huang, 1999; Gneiting, 2002). What distinguishes half-spectral approaches from other modeling methods is that valid dependence models can be defined

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using both spectral and non-spectral elements. Notably for this paper, half-spectral approaches can also enable direct modeling of space–time interaction. Details and definitions of half-spectra and their formulations follow in Section 2.1.

The investigation in this paper is distinguished from other articles by our focus on developing the space–time interaction properties of the models we present. We believe this paper is novel in this regard. Commonly, models that may be factored into purely spatial and purely temporal components – often referred to as separable models – are avoided (Kyriakidis and Journel, 1999). Similarly, models with non-differentiable ridges or kinks in the covariance function have been shown to have undesirable properties (Stein, 2005, 1999). In this paper we give positive and specific recommendations in terms of space–time interaction, and we build a class of space–time models that meets these recommendations.

The space–time interaction behavior we seek is motivated by screening effect considerations (Chilès and Delfiner, 1999). A screening effect is an intuitively desirable property of some covariance models that implies that information at a target point in a random field is asymptotically completely determined by other points in the target point's local neighborhood. An overview of the screening effect and our complete analysis are in Sections 2.2 and 3. To summarize, we develop our models by following a condition developed in Stein (2010) that guarantees a screening effect for non-differentiable models. Though this condition guarantees a screening effect in only some models, Stein (2010) also suggests it may lead to some generally desirable model behavior as it will limit irregularities in the high-frequency components of a model.

The class of models we develop in this paper is a set of stationary non-separable space–time dependence models. This class allows for nearly arbitrary marginal temporal dependence modeling while at the same time is Matèrn-like marginally in space. In this way, arbitrarily and independently, different smoothnesses may be specified in time and space in this class. This class of models furthermore satisfies the screening effect condition in Stein (2010) under mild conditions on the marginal temporal spectrum.

Outside of half-spectral approaches, there are a host of techniques that can be used to produce valid non-separable space–time and other multidimensional covariance models. Convolution and mixtures methods provide a wide range of solutions (Brown et al., 2000; Ma, 2003; Porcu et al., 2010; Kolovos et al., 2004; Fonseca and Steel, 2011). Full spectral methods also give tremendous flexibility to produce stationary (Kolovos et al., 2004; Stein, 2005; Fuentes et al., 2007) and non-stationary models (Porcu et al., 2009). Genton (2007) approximates non-separable models using separable covariances. Also, expanding on the popular approach in Gneiting (2002), Zastavnyi and Porcu (2011) provide characterization theorems and necessary conditions to produce space–time models with compact support, and Porcu et al. (2006) create space–time models focusing on spatial anisotropy.

This paper is structured as follows. Section 2 introduces the theory and definitions of the half-spectral approach and describes in more detail the screening effect condition in Stein (2010). Section 3 develops theory of certain half-spectral models with respect to the condition in Stein (2010). Basic properties of the half-spectral models we develop are given, and we develop two necessary conditions for models to meet the screening condition. Section 4 presents the new class of models that meets the restrictions developed in Section 3. In Section 5, we fit examples of models from this class to the frequently used Irish wind dataset, and we compare these models to other separable and non-separable models.

2. Half-spectrum definitions and the screening effect condition

2.1. Half-spectra

All continuous stationary covariance functions, K, for a real valued process on $\mathbb{R}^m$ can be written in the form $K(x) = \int \exp(i\xi^T x)F(d\xi)$ for F a symmetric finite positive measure on $\mathbb{R}^m$ and $\xi \in \mathbb{R}^m$. If f exists such that $F(d\xi) = f(\xi)d\xi$, f is called the spectral density.

For a stationary, continuous space–time covariance model $K(s, t)$ with $s \in \mathbb{R}^d$ and $t \in \mathbb{R}$ with integrable spectral density, $g(\lambda, \omega)$, where $\lambda \in \mathbb{R}^d$ and $\omega \in \mathbb{R}$, we can represent K with g via Fourier