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Tukey max-stable processes for spatial extremes

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ABSTRACT

We propose a new type of max-stable process that we call the Tukey max-stable process for spatial extremes. It brings additional flexibility to modeling dependence structures among spatial extremes. The statistical properties of the Tukey max-stable process are demonstrated theoretically and numerically. Simulation studies and an application to Swiss rainfall data indicate the effectiveness of the proposed process.

1. Introduction

Max-stable processes are widely used to model spatial extremes because they naturally arise as the limits of pointwise maxima of rescaled stochastic processes (de Haan and Ferreira, 2006) and therefore extend generalized extreme-value distributions to that setting; see the reviews of recent advances in the statistical modeling of spatial extremes by Davison et al. (2012), Cooley et al. (2012), Ribatet (2013) and Davison and Huser (2015). A useful spectral characterization of simple max-stable processes for which the margins are standardized to unit Fréchet distributions was proposed by de Haan (1984), Penrose (1992) and Schlather (2002). Specifically, let $\{R_j\}_{j=1}^{\infty}$ be the points of a Poisson process on $(0, \infty)$ with intensity $r^{-2}dr$, and let $\{W_j(s)\}_{j=1}^{\infty}$ be independent replicates of a nonnegative stochastic process, $W(s)$, on $\mathbb{R}^d$ with continuous sample paths satisfying $\mathrm{E}\{W(s)\} = 1$ for all $s \in \mathbb{R}^d$.

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Then,

$$Z(s) = \max_{j \geq 1} \{R_j W_j(s)\} \tag{1}$$

is a max-stable process on $\mathbb{R}^d$ with unit Fréchet marginal distributions; that is, $\text{pr}\{Z(s) < z\} = \exp(-1/z)$ for all $z > 0$ and $s \in \mathbb{R}^d$. The finite-dimensional distributions of simple max-stable processes follow from (1) for any set of D locations, $s_1, \ldots, s_D$:

$$\text{pr}\{Z(s_1) \leq z_1, \ldots, Z(s_D) \leq z_D\} = \exp\{-V_{s_1,\ldots,s_D}(z_1, \ldots, z_D)\}, \quad z_1, \ldots, z_D > 0, \tag{2}$$

where the exponent measure

$$V_{s_1,\ldots,s_D}(z_1, \ldots, z_D) = \text{E}\left[\max_{i=1,\ldots,D} \left\{\frac{W(s_i)}{z_i}\right\}\right]$$

captures all the extremal dependence information. The latter can be summarized by the extremal coefficient function (Schlather and Tawn, 2003):

$$\theta_D(s_1, \ldots, s_D) = V_{s_1,\ldots,s_D}(1, \ldots, 1) = \text{E}[\max_{i=1,\ldots,D} \{W(s_i)\}] \in [1, D],$$

where the values 1 and D for θ_D correspond to complete dependence and independence, respectively. For a pair of locations (s_1, s_2), under the stationarity assumption of the latent Gaussian random field $W(s)$, the bivariate extremal coefficient function $\theta_2(s_1, s_2) \equiv \theta(h)$ with $h = \|s_1 - s_2\|$.

Different choices for $W(s)$ in (1) lead to various models for spatial max-stable processes proposed in recent years. In a seminal unpublished University of Surrey 1990 technical report, R. L. Smith proposed a Gaussian extreme-value process based on $W(s) = \phi_d(s - U; \Sigma)$, where $\phi_d(\cdot; \Sigma)$ denotes the d-dimensional Gaussian probability density function with covariance matrix Σ and U is a unit rate Poisson process on $\mathbb{R}^d$. Schlather (2002) considered the extremal-Gaussian process obtained with $W(s) = (2\pi)^{1/2} \max\{0, \varepsilon(s)\}$ where $\varepsilon(s)$ is a stationary standard Gaussian process with correlation function $\rho_\Psi(h)$ and parameters Ψ. A generalization called the extremal-t process (Nikoloulopoulos et al., 2009; Opitz, 2013) was defined by taking $W(s) = \pi^{1/2} 2^{1-\nu/2} [\Gamma\{(\nu + 1)/2\}]^{-1} \max\{0, \varepsilon(s)\}^\nu$ where $\Gamma(\cdot)$ is the gamma function and $\nu > 0$ the degrees of freedom. The case $\nu = 1$ reduces to the extremal-Gaussian process. The pairwise extremal coefficient for the extremal-t process is

$$\theta_t(h) = 2T_{\nu+1}\left\{(\nu + 1)^{1/2}\sqrt{\frac{1 - \rho_\Psi(h)}{1 + \rho_\Psi(h)}}\right\},$$

where T_ν denotes the cumulative distribution function of a standard Student-t random variable with $\nu > 0$ degrees of freedom. It is trivial to see that $\theta_t(h) \leq \theta_t^0$ with $\theta_t^0 = 2T_{\nu+1}\{(\nu + 1)^{1/2}\}$ and that as $\rho_\Psi(h) \to 0$, $\theta_t(h) \to \theta_t^0$. The θ_t^0 can only approach 2, which corresponds to independence, when $\nu \to \infty$, although this can be resolved by incorporating a compact random set element (Schlather, 2002; Davison and Gholamrezaee, 2012; Huser and Davison, 2014). Modeling of extreme values with asymptotically independent processes has been discussed by de Haan and Zhou (2011), Wadsworth and Tawn (2012) and Padoan (2013).

Another popular spatial max-stable model is the so-called geometric Gaussian process for which $W(s) = \exp\{\sigma\varepsilon(s) - \sigma^2/2\}$ with some $\sigma > 0$. The resulting extremal coefficient is

$$\theta_{geo}(h) = 2\Phi\left\{\sigma\sqrt{\frac{1 - \rho_\Psi(h)}{2}}\right\}, \tag{3}$$

where Φ denotes the cumulative distribution function of a standard Gaussian random variable. Similar to the extremal-t process, $\theta_{geo}(h)$ is bounded above by $\theta_{geo}^0 = 2\Phi\left(\sigma/\sqrt{2}\right) < 2$ and the upper-bound $\theta_{geo}^0 \to 2$ only if $\sigma \to \infty$. The geometric Gaussian process can be viewed as a special case of the well-known Brown–Resnick process (Brown and Resnick, 1977; Kabluchko et al., 2009), where the latent process is defined as $W(s) = \exp\{\tilde{\varepsilon}(s) - \gamma(s)\}$ with $\tilde{\varepsilon}(s)$ being an intrinsically stationary Gaussian