Spatial Statistics 18 (2016) 40–53

Contents lists available at ScienceDirect

Spatial Statistics

journal homepage: www.elsevier.com/locate/spasta

Monte Carlo testing in spatial statistics, with applications to spatial residuals

Tomáš Mrkvička [a,*], Samuel Soubeyrand [b], Mari Myllymäki [c], Pavel Grabarnik [d], Ute Hahn [e]

[a] *Department of Applied Mathematics and Informatics, Faculty of Economics, University of South Bohemia, České Budějovice, Czech Republic*

[b] *BioSP, INRA, 84914, Avignon, France*

[c] *Natural Resources Institute Finland (Luke), P. O. Box 18, FI-01301 Vantaa, Finland*

[d] *Institute of Physico-Chemical and Biological Problems in Soil Science, the Russian Academy of Sciences, Pushchino, Russia*

[e] *Centre for Stochastic Geometry and Advanced Bioimaging, Department of Mathematics, Aarhus University, 8000 Aarhus C, Denmark*

ARTICLE INFO

Article history:
Received 14 December 2015
Accepted 14 April 2016
Available online 27 April 2016

Keywords:
Deviation measure
Global envelope test
Goodness-of-fit test
Multiple testing problem
Rank envelope test
Residuals

ABSTRACT

This paper reviews recent advances made in testing in spatial statistics and discussed at the Spatial Statistics conference in Avignon 2015. The rank and directional quantile envelope tests are discussed and practical rules for their use are provided. These tests are global envelope tests with an appropriate type I error probability. Two novel examples are given on their usage. First, in addition to the test based on a classical one-dimensional summary function, the goodness-of-fit of a point process model is evaluated by means of the test based on a higher dimensional functional statistic, namely a two-dimensional smoothed residual field. Second, a goodness-of-fit test of a geostatistical model is performed based on two-dimensional raw residuals.

* Corresponding author.
E-mail addresses: mrkvicka.toma@gmail.com (T. Mrkvička), samuel.soubeyrand@avignon.inra.fr (S. Soubeyrand), mari.myllymaki@luke.fi (M. Myllymäki), gpya@rambler.ru (P. Grabarnik), ute@imf.au.dk (U. Hahn).

http://dx.doi.org/10.1016/j.spasta.2016.04.005
2211-6753/

1. Introduction

In spatial statistics, hypothesis tests are essential steps in data analysis. Most often goodness-of-fit tests are performed in order to check the compatibility of a fitted model with the data. Also other kinds of hypotheses are studied in which e.g. sets of point patterns, random fields or random sets are compared, or independencies of marks and points in point patterns or random superposition of patterns of points with different marks in multi-type point patterns are tested.

For the purpose of testing, the information of the spatial processes must be summarized in a reasonable way. Most often, test functions $T(r)$ which depend on distance r (between two locations or points) are employed for this purpose. Examples are estimators of well-known summary functions such as the variogram, cross-variogram, Ripley's K-function, mark-weighted K-function, multitype K-function or the empty space function. In recent years, functions of the spatial position, such as residual fields for spatial point patterns, have become popular for exploratory analysis. In Section 5, we demonstrate how testing carries over to such functions with arguments in $\mathbb{R}^d$.

Usually, the distributions of the test functions under the null hypotheses are not known explicitly. Therefore, one resorts to the Monte Carlo method, comparing the observed estimated function $T_1(r)$ to the distribution of $T(r)$ under the null hypothesis, for r-values in some pre-chosen interval $I \subset \mathbb{R}^d$. Different strategies to make this comparison have been suggested in literature.

A very popular approach to checking the compatibility between data and a proposed model is based on the so-called *conventional envelope method* (Ripley, 1977; Besag and Diggle, 1977), where s simulations of the test function under the null model, $T_2(r), \ldots, T_{s+1}(r)$, are produced and the lower envelope $T_{\text{low}}(r) = \min_{i=2,\ldots,s+1} T_i(r)$ and the upper envelope $T_{\text{upp}}(r) = \max_{i=2,\ldots,s+1} T_i(r)$ are computed. Finally T_1 is compared with these envelopes. This procedure has become widely used, since it gives a graphical interpretation indicating the distances where the data function is not in accordance with the null model, which information is important to understand reasons for rejection and to seek for alternative models.

This procedure can, however, be used only as an exploratory tool (Baddeley et al., 2014): the functions are inspected at many distances of the interval I simultaneously, whereas the type I error of the test is controlled for a fixed $r \in I$ only. Inspecting the functions on I leads to a serious multiple testing problem. This was mentioned already by Ripley (1977), but Loosmore and Ford (2006) demonstrated that the type I error probability of this procedure can be unacceptably high. To express the local significance of the method, this method is often called *the pointwise envelope* (Baddeley et al., 2014).

While Loosmore and Ford (2006) advised to avoid the envelope method for testing and recommended to use instead deviation tests (Diggle, 1979), Grabarnik et al. (2011) showed that the envelope test can be refined to a rigorous statistical tool.

The deviation test has been a popular way to handle the multiple testing problem. This test and its advantages and disadvantages are recalled in Section 2. A global envelope test is a solution that addresses shortcomings of both the pointwise envelope and the deviation test. It is a statistical test that rejects the null hypothesis if the observed function T_1 is not completely inside a global envelope given by T_{low} and T_{upp}, i.e. it rejects if there exists $r \in I$ such that $T_1(r) \notin (T_{\text{low}}(r), T_{\text{upp}}(r))$. A global envelope test with appropriate type I error probability, namely the rank envelope test (Myllymäki et al., 2016), is presented in Section 3.

Section 4 further gives answers to the most important issues connected with the usage of the rank envelope test. Finally, Sections 5 and 6 present examples from point pattern analysis and geostatistics, respectively. In the former section, both one- and two-dimensional test functions are used and in the latter, the test function is two-dimensional, representing spatial residuals of a fitted model.

The proposed methods are provided in the R library *spptest*, which can be obtained at https://github.com/myllym/spptest.

2. Deviation tests

The deviation test overcomes the multiple testing problem by summarizing the information contained in $T(r)$, $r \in I$, into a single number, u, and performing a univariate Monte Carlo test (Illian