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Three-dimensional thermo-elastoplastic analysis of thick functionally graded plates using the meshless local Petrov–Galerkin method

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ABSTRACT

A numerical method based on the meshless local Petrov–Galerkin (MLPG) method is presented for three-dimensional (3D) thermo-elastoplastic analysis of thick functionally graded (FG) plates subjected to combined thermal and mechanical loads. The FG plate is assumed to be made of two constituents, whose volume fractions vary continuously in the thickness direction according to a power law. All material properties are considered to be temperature dependent. The von-Mises yield criterion and isotropic strain hardening rule are employed to describe the elastoplastic behaviors of the FG plates. The weak form is derived using the 3D equilibrium equations, and then it is transformed into local integral equations on brick-shaped local sub-domains by using a Heaviside step function as the test function. The proposed approach makes it possible to distribute more nodes in the direction of the material variation to construct the shape and test functions. Consequently, more accurate solutions can be obtained easily and effectively. Several numerical examples for temperature, displacement and stress analysis of thick FG plates are presented for different material gradients and boundary conditions. The obtained results have been compared with accurate finite element results and an excellent agreement has been observed.

1. Introduction

In recent years, heterogeneous materials such as functionally graded materials (FGMs), which are suitable for high-temperature environment, have been used in many advanced structures. Actually, the FGMs are new composites in which the material properties smoothly and continuously vary by a specific function. Usually, the FGMs are composed of ceramics and metals so that the material properties vary from the metallic side to the ceramic one. The FGMs were initially used as thermal barrier materials for aerospace structural applications and fusion reactors. Now they are employed for the general use as structural components in high-temperature conditions. The most important advantage of the FGMs is that they are able to resist high temperature conditions while keeping their strength.

Investigation on the thermo-mechanical behavior of FGMs is an active research area in engineering mechanics. Researchers have studied the thermo-mechanical behavior of many different members made of FGMs. Among the studied members, the plates are the most notable because of their wide applications in modern engineering. In a FG plate, the volume fraction of the constituent materials changes gradually, in the thickness direction.

Various assumptions are considered to formulate a plate as a 2D problem. For this purpose, different 2D theories, such as the classical plate theory (CPT) or the shear deformation plate theories (SDTs) are introduced. Although, finding a solution for the plate in 2D formulation is easier, but the simplifying assumptions considered in 2D theories cause some errors in the solutions. Also, by increasing the thickness of the plate, the error increases and therefore, it may lead to unreliable results. Clearly, if 3D solutions are achievable, it will be more realistic and accurate than the solutions obtained by the 2D theories and the 3D solutions allow further physical insights [1].

Several analytical methods have been used for 3D analysis of FG plates. An exact 3D solution for laminated composite and FG plates with traction-free surfaces were provided by Mian and Spencer [2]. Reddy and Cheng [3] employed the asymptotic expansion method to derive analytical solutions for 3D analysis of simply supported FG plates under thermal loadings. An exact solution for the 3D thermo-elastic deformations of a simply supported FG plate with a power-law variation of the volume fractions of the constituents was established by Vel and Batra [4]. They also studied transient heat conduction problems of simply supported FG plates subjected to either time-dependent heat flux or temperature on the top and bottom surfaces of the plate [5]. Kashtalyan [6] developed a 3D elasticity solution for the bending analysis of simply

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supported FG plates subjected to a transverse loading. Specific variations of material properties such as exponential model, linear model and reciprocal model were considered for the 3D analysis of simply supported FG plates by Zhong and Shang [7]. Although analytical methods can provide closed-form solutions, they are restricted to simple geometries, specific types of grading of material properties, certain types of boundary conditions and some special cases of loading. Consequently, approximate solutions of realistic and practical engineering problems are usually achieved by numerical methods.

Various numerical methods such as the finite element method (FEM), the boundary element method (BEM) and the differential quadrature (DQ) method are the alternatives that have been used for analysis of practical problems in science and engineering. There are also some other discretization techniques such as the methods based on isogeometric analysis [8–10], the non-uniform rational B-sline (NURBS)-based FEM [11], and other FEM-based algorithms [12] for analysis of structural problems. In recent years, meshless methods which do not require serious effort for mesh generation have attracted much attention. A variety of these methods, such as the element-free Galerkin (EFG) method [13], the reproducing kernel particle method (RKPM) [14], the hp-clouds [15], the partition of unity method (PUM) [16], the diffuse element method (DEM) [17], the natural element method (NEM) [18], the immersed particle method [19] and some other alternative methods [20,21], have been successfully employed in analysis of different engineering problems. However, most of these methods need background cells/mesh for evaluation of domain integrals [22] in the weak form of governing equations and therefore they are not a truly meshless method.

The MLPG method, provided by Atluri and Zhu [23], Atluri et al. [24], and Atluri and Shen [25,26] considers local weak form and does not need a background mesh for evaluation of the integrals in the weak form of the problem. In the MLPG method, the integrals of the weak form are computed over local sub-domains that partially cover each other. The trial and test functions are selected from completely different functional spaces. Also, the test and trial domains can have different physical size, which makes the MLPG a very flexible method. According to the concept of the MLPG, six different methods have been introduced, which are labeled as MLPG1–MLPG6 [26]. Difference between these six methods is caused by the type of the test functions considered in the weak formulation. Among them, the MLPG5; wherein the Heaviside step function is employed as a test function; eliminates the necessity of domain integration, and shows high robustness and accuracy [25]; so it is used in the present study. The MLPG methods have been applied in a broad range of applications, e.g. elasto-statics [27], elasto-dynamics [28], thermoelasticity [29], elasto-plasticity problem [30,31], beam problems [32,33], plate problems [34,35] and FGM problems [36–42]. Up to now, the applications of the MLPG method have been mostly restricted to 2D problems. The main reason for this fact refers to difficulties in evaluating the integrals over the local sub-domains for the 3D analysis, especially when a sub-domain intersects the global boundary of the problem [43].

However, impressive efforts have been carried out to apply the MLPG method to solve 3D problems. A combination of MLPG2 and MLPG5 for the solution of two classical 3D problems, i.e., the Boussinesq problem and the Eshelby's inclusion problem was provided by Li et al. [44]. Han and Atluri [45] used the MLPG method to solve 3D elastic fracture problems. They examined the efficiency of three kinds of the MLPG methods by different test functions for analysis of thick beams and spheres [46]. Also, Han and Atluri [47] employed the MLPG domain discretization method to solve 3D elasto-dynamic problems of impact and fragmentation. Two different 3D MLPG procedures including MLPG1 and MLPG5 for the elasto-static analysis of thick plates with various boundary conditions were developed by Vaghefi et al. [43]. MLPG method has also been used to analyze deformations of engineering structures composed of FGMs. The MLPG5 method for the elastic analysis of 3D cube and hollow cylinder in anisotropic FGMs was developed by Sladek et al. [48]. Rezaei Mojdehi et al. [49] analyzed a 3D dynamic problem of thick FG plates using the MLPG5 method. In order to model material variation, more nodes were used in the thickness direction of the plate. Vaghefi et al. [50] provided a 3D elasto-static solution for thick FG plates with various boundary conditions by utilizing two different MLPG procedures including MLPG1 and MLPG5. An exponential function was assumed for the variation of Young's modulus through the thickness of the plate, while the Poisson's ratio was assumed to be constant. Most of the presented MLPG methods are related to elastic analysis and, to the authors' best knowledge, the MLPG method has not been developed yet for 3D thermo-elastoplastic analysis of thick FG plates.

In this paper, a complete 3D MLPG method is developed for the thermo-elastoplastic analysis of thick FG rectangular plates with various boundary conditions, subjected to combined thermal and mechanical loads. The FG plate is assumed to be composed of two materials whose volume fractions vary continuously in the thickness direction according to a power law. All material properties such as thermal, elastic and plastic properties are assumed to be temperature dependent. Effective elastic and plastic material properties, such as the elastic modulus, the initial yield stress, and the tangent modulus, are modeled by the modified rule of mixtures [51]. The uncoupled nonlinear thermo-elastoplastic formulation is considered. The small strain increment theory is adopted and von-Mises yield function with isotropic linear hardening rule is considered to describe the elastoplastic behaviors of the FG plates. Plastic strain increment is given by the Prandtl–Reuss rule. The local weak form is used to formulate the problem and the 3D MLS approximation is utilized to approximate the field variables. The penalty method is adopted to impose the essential boundary conditions. Brick-shaped regions are considered as subdomains and support domains for evaluation of the integrals of the weak form and approximating the solution variables, respectively. The proposed approach makes it possible to add the nodes in any direction of the plate depending upon the required accuracy. Therefore, adequate number of nodes has been distributed in the thickness direction of the plate to approximate the transient temperature, displacement and stress variations through the thickness with a good accuracy.

In order to demonstrate the validity and efficiency of the proposed method, several numerical examples are presented and the results are compared with those from the finite element method. Also, the convergence of the proposed method with respect to the number of nodes is investigated. It is observed that highly accurate results can be obtained by the developed MLPG method for analysis of 3D thermo-elastoplastic problems.

2. Basic equations

In this part, the governing equations and boundary conditions of nonlinear transient heat conduction and elastoplastic problems are described.

2.1. Transient heat conduction in non-homogenous domains with temperature dependent material properties

The 3D transient heat conduction equation in the non-homogeneous isotropic domain Ω with the surface Γ, in the absence of