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A meshless solution of two dimensional multiphase flow in porous media

Karel Kovářík*, Soňa Masarovičová, Juraj Mužík, Dana Sitányiová

University of Žilina, Faculty of Civil Engineering, Department of Geotechnics, Univerzitná 8215/1, 010 26 Žilina, Slovakia

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ABSTRACT

Multiphase fluid flow problems are of importance in many disciplines including hydrology and petroleum reservoir engineering. Standard methods such as the finite differences, finite volumes and expanded mixed finite elements methods use very general unstructured grids and need different grid adaptation strategies to ensure optimal solution of this non-linear problem. The meshless methods seem to be quite a good alternative to these classical mesh-based methods. In our work we used the meshless Petrov–Galerkin local method based on the pressure-saturation formulation.

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1. Introduction

A multiphase flow appears mainly in problems related to the environment and the energy. This paper is focused on the modelling of two-phase flow, for example the flow of a wetting phase like the groundwater and a non-wetting phase like dense non-aqueous liquids through the porous medium. The problem is non-linear and therefore the simulation requires usually large meshes and also too much computational time even for simulations of testing examples. Typical numerical methods used to solve these problems are based on different formulations of the finite differences, volume and element methods [1,2] or the discontinuous Galerkin method [3,4].

Meshless methods are widely used in the last decades due to their flexibility in solving various boundary value problems and possibility to reduce a problem with generation of different meshes. Therefore these methods are considered as a powerful approach to solve partial differential equations of various kinds. The large number of meshless methods have been developed by different authors as several types of least square collocation meshless method [5–9], meshless local Petrov–Galerkin method (MLPG) [10,11], local boundary integral element method (LBIEM) [11], radial basis integral equation method (RBIEM) [12], etc. The least square collocation methods require no integration but they have deficiency with formulation of boundary conditions and singularities as pumping wells. The MLPG, LBIEM, and RBIEM are the local weak methods and they can easy deal with different boundary conditions but evaluation of integrals is needed.

In our paper we try to present a meshless numerical method based on local Petrov–Galerkin formulation (MLPG). This method uses a local symmetric weak form to solve the problem of multiphase flow. The most important advantages of this method are simple computation of all needed integrals as they are regular and also very easy setting of boundary conditions of the second kind. This property results from the weak formulation of the solved problem.

The MLPG method has been introduced by Atluri et al. [10,11]. It is characterized as meshless since distributed nodal points, covering the domain, are employed. These nodal points can be randomly spread over the domain but it is well-known that using completely randomly distributed nodes may lead to less accurate results [13]. Therefore a certain effort should be invested into the positioning of the points or more sophisticated algorithms for selection neighbourhood nodes used for interpolation can be also used [14]. All needed integrals are carried out on the local subdomain centred at every point. All unknown variables are approximated by some interpolation method to obtain a system of non-linear equations. Solving this system of equations leads to a numerical solution of the problem. Atluri et al. [10] used the moving least squares (MLS) approximation scheme but nowadays the radial basis functions (RBFs) interpolation can be used instead (see e.g. [15,16]). An important advantage of RBF interpolation is an existence of the delta property and therefore the boundary conditions of the first kind can be easily defined.

In this paper the solution of two-phase flow through porous medium based on the MLPG-RBF method is presented. Our main

* Corresponding author.
E-mail address: kovarik@fstav.uniza.sk (K. Kovářík).

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goal was to investigate the robustness and the ability of this meshless method to solve this non-linear and heterogeneous problems.

2. Governing equations of multiphase flow

A two-phase flow through porous media can be usually described by the mass balance equation and Darcy's law for each of the fluid phases (see also [3])

$$\frac{\partial\left(\rho_\alpha \phi S_\alpha\right)}{\partial t} + \nabla.\left(\rho_\alpha v_\alpha\right) = \rho_\alpha f_\alpha \tag{1}$$

$$v_\alpha = -\frac{K k_{r\alpha}}{\mu_\alpha}\left(\nabla p_\alpha - \rho_\alpha g\right) \tag{2}$$

where $\alpha = w$ is the wetting phase (e.g. water), $\alpha = n$ is the non-wetting phase (e.g. oil or air), ϕ and K are the porosity and the absolute permeability of the porous media. S_α, p_α, v_α, ρ_α, μ_α and $k_{r\alpha}$ are, respectively, the saturation, pressure, volumetric velocity, density, viscosity and the relative permeability of the α-phase.

In addition to Eqs. (1) and (2) the following relations should be also fulfilled

$$S_w + S_n = 1 \quad p_c = p_n - p_w \tag{3}$$

where p_c is the capillary pressure.

To simplify the Darcy's law equation (2) we define phase mobilities

$$\lambda_\alpha = \frac{k_{r\alpha}}{\mu_\alpha}, \quad \alpha = w, n \tag{4}$$

and (2) can be written as

$$v_\alpha = -\lambda_\alpha K\left(\nabla p_\alpha - \rho_\alpha g\right) \tag{5}$$

We have focused on the incompressible fluid flow, i. e. the densities ρ_α are constant. Furthermore, we assume that the porosity ϕ remains constant over the whole domain of interest. Then Eq. (1) can be simplified

$$\phi\frac{\partial S_\alpha}{\partial t} + \nabla.\, v_\alpha = f_\alpha \tag{6}$$

The mass balance equation (6) and the Darcy's law (5) is the basis to description the multiphase incompressible flow. The pressure and saturation can be coupled using (3). Several formulations of the two-phase flow problem are possible (see e.g. [3] or [17]). In our paper we focused on the formulation based on the saturation and pressure of the wetting phase (S_w and p_w).

$$-\nabla\left[\lambda_t K \nabla p_w + \lambda_n K \nabla p_c - \left(\rho_w + \rho_n\right) g\right] = f_w + f_n \tag{7}$$

$$\phi\frac{\partial S_w}{\partial t} - \nabla\left(\lambda_w K \nabla p_w - \rho_w g\right) = f_w$$

where λ_t is the total mobility defined as $\lambda_t = \lambda_w + \lambda_n$. These two equations are coupled because the mobilities and the capillary pressure are functions of the effective wetting phase saturation S_e defined as

$$S_e = \frac{S_w - S_{rw}}{1 - S_{rw} - S_{rn}} \tag{8}$$

where S_{rw} and S_{rn} are residual wetting and non-wetting phase saturations. In our paper the Brooks–Corey model [18] is considered

$$k_{rw} = S_e^{\frac{2+3m}{m}} \quad k_{rn} = (1 - S_e)^2\left(1 - S_e^{\frac{2+m}{m}}\right) \tag{9}$$

where m is the dimensionless pore size distribution index. The capillary pressure is defined as

$$p_c = p_d S_e^{-\frac{1}{m}} \tag{10}$$

where p_d is a constant entry pressure.

3. Meshless local Petrov–Galerkin formulation of the problem

The entire domain Ω is covered by nodes located inside the area and also on the global boundary Γ (see Fig. 1). The local weak formulation of the multiphase flow is formulated over a local sub-domain, created around every node. This sub-domain can be any simple geometry (rectangular or circle in 2D).

The mutual relationship of particular nodes is based on some interpolation algorithm. The local radial basis functions (RBFs) are used to approximate unknown pressure and saturation of the wetting phase p_w and S_w in the neighbourhood or support of a reference point i. Multiquadric functions are one of the most popular radial functions used for this purpose and they have been used in our paper. They can be defined as

$$R(r_{ij}) = \sqrt{r_{ij}^2 + \epsilon^2} \tag{11}$$

where r_{ij} is a distance between points i and j and ϵ is the so-called shape factor of multiquadric function. The formula of Hardy [19] with a slight modification is applied to the local RBFs (see [15]) to find the optimal value of the shape factor, which can be computed in point i as

$$\epsilon = \frac{0.815}{N}\sum_{j=1}^{N} r_{ij} \tag{12}$$

The interpolation of the unknown variables can be written using the basis functions φ_{ij} in the form (more details can be found in e.g. [16] or [15])

$$p_{wi} = \sum_{j=1}^{N}\varphi_{ij} p_{wj}, \quad S_{wi} = \sum_{j=1}^{N}\varphi_{ij} S_{wj} \tag{13}$$

where N is the number of neighbourhood points. There are several

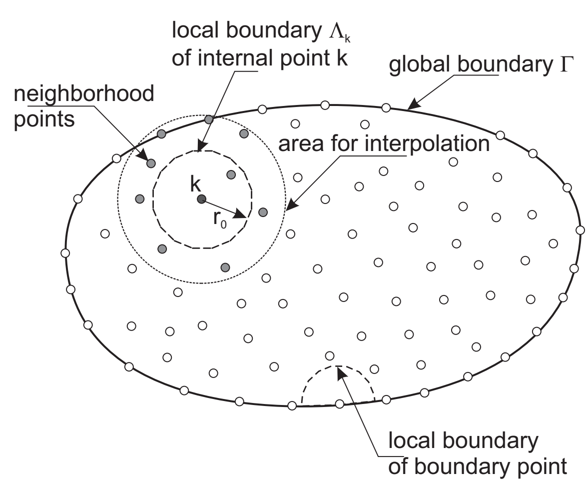

Fig. 1. Points in the global area.

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