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A boundary element and level set based topology optimisation using sensitivity analysis

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ABSTRACT

The structural topology optimisation method presented in this paper is based on the boundary element method, level set method and shape sensitivity analysis for two-dimensional linear elastic problems. The proposed method automatically nucleates holes within the design domain during the optimisation process using a topological derivative based hole insertion criterion. The level set method is used to provide an implicit description of the structural geometry, which is capable of automatically handling topological changes, i.e. holes merging with each other or with the boundary. During the optimisation process non-uniform rational b-splines are fitted through the zero level set contours, which links an implicit geometry representation to its structural model. In addition, this provides an optimal design in standard CAD format, and without intermediate material densities, which can be directly used in other design processes. The proposed optimisation method is tested against different benchmark examples and the optimal geometries generated are in close agreement those available in the literature of topology optimisation.

1. Introduction

The level set method (LSM) is an efficient numerical technique originally developed by Osher and Sethian [1] for the tracking of propagating interfaces with topological changes of merging and breaking naturally. There is a wide variety of applications, including structural optimisation, in which LSM has been successfully implemented. Sethian and Wiegmann [2] first presented a level set (LS) based structural optimisation method. In their implementation, shape and topology changes were accomplished through a von Mises stress based criterion. Osher and Santosa [3] proposed an LS based method using shape sensitivity analysis for the optimisation of an inhomogeneous drum for the frequency response. Wang et al. [4] also presented a shape sensitivity approach for the solution of minimum compliance problems. Allaire et al. [5] independently proposed an LS based optimisation method based on shape sensitivities for the solution of 2D and 3D optimisation problems with both linear and non-linear structural materials.

In an LS based optimisation approach, the selection of an effective structural performance measuring tool plays an important role for the solution of optimisation problems. The performance measuring tool predicts the structural response against the applied load and boundary conditions. These responses are then converted into a useful form through shape sensitivity analysis, which informs the evolution of the structural geometry accordingly. The performance of a candidate design can be measured through a geometry mapping technique, which projects the implicitly represented geometry onto the structural model. The most commonly used geometry mapping techniques in the LS based structural optimisation are material distribution (density based), immersed boundary and conforming discretisation [6].

Due to a continuously evolving geometry the standard finite element method (FEM) without re-meshing is not recommended as a structural performance measuring tool in structural optimisation. Therefore, most of the LS based optimisation methods utilise a fixed Eulerian type mesh with an "Ersatz material" approach [5] as an alternative finite element (FE) analysis tool. The structural geometry is represented through a density distribution function, i.e. $(\eta < \rho < 1)$ similar to the density based optimisation approach [7]. Solid material is represented by $(\rho = 1)$ and holes in the structure are replaced by a specified minimum relative density $(\rho = \eta)$. Wang et al. [4] and Allaire et al. [5] initially implemented the density based approaches in their proposed LS based topology optimisation methods. Although the fixed grid is a simple approach, it is not effective to capture the exact geometry of the boundary [5] and a highly dense grid distribution is always required near the boundary for high accuracy [8]. In addition, the presence of intermediate material densities along the structural boundary can result in non-smooth and indistinct boundary

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representation [9]. A smoothed Heaviside function approach has been adopted to smooth the discontinuity at the boundary [10,11]. However, the numerical integration of the stiffness matrix may be less accurate [12].

The second type of geometry mapping is based on the immersed boundary approach, which uses a non-body conforming fixed grid. Therefore, the structural geometry is not aligned with the grid and can intersect some grid cells. This approach allows a clear boundary representation and avoids intermediate density material [6]. Sethian and Wiegmann [2] used the immersed interface method within a finite difference framework for the solution of LS based topology optimisation problems. The extended finite element method (X-FEM) has also been used to evaluate the required properties at the structural boundary through the local enrichment of elements intersected by the zero level set contour [13]. Belytschko et al. [14] combined the implicit boundary representation with the X-FEM approach for the solution of topology optimisation problems. The X-FEM has also been used in the LS based optimisation methods presented in [15,16]. Yamasaki et al. [9] developed a two-dimensional topology optimisation method for minimum compliance problems based on the immersed boundary mapping, boundary element and level set methods. The common problem reported in the implementation of immersed boundary methods is the occurrence of small intersection of finite elements [15] or short boundary elements [9] while discretising the structural model. This can profoundly affect the accuracy of structural response. Further, the use of immersed boundary techniques requires sophisticated codes and can make their implementation difficult and time consuming [6].

Some of the LS based optimisation methods use two types of meshes during the numerical implementation, i.e. a fixed Eulerian mesh which maintains the LS function throughout the optimisation process, and a second mesh which exactly fits the design domain. Two different approaches can be used to discretise the design domain, i.e. the domain discretisation (i.e. the FEM) and boundary only discretisation (i.e. the BEM). This third type of mapping provides the most accurate analysis of the structural model and especially along the boundary. The use of BEM with the level set method in two-dimensional structural optimisation was first used by Abe et al. [17] for the solution of minimum compliance problems. The proposed approach has also been extended for shape optimisation of sound scattering problems [18]. The use of BEM for acoustic applications has also been thoroughly investigated in the research work presented in [42–44]. In those research studies the topological sensitivities are formulated through the BEM framework accelerated with the fast multipole method. In the research work of Isakari et al. [45] a topology optimisation method was presented through the integration of LSM, fast multipole boundary element method and topological sensitivity analysis. The proposed method was applied to three-dimensional wave scattering problems. Ha and Cho [19] utilised an unstructured domain conforming discretisation approach for the optimisation of geometrically nonlinear structures within the LS framework. Yamasaki et al. [20] presented a boundary tracking approach for the LS based topology optimisation using a conforming discretisation approach and geometry based re-initialisation scheme [21].

In comparison with the immersed boundary mapping, the body conforming approach is attractive due to its simplicity and higher accuracy. However, the domain discretisation based body conforming mapping, i.e. FEM requires special care for a continuously changing structural geometry; that it is difficult to ensure the accuracy of analysis for a continuously changing FE model. However, the boundary based body mapping, i.e. the BEM is attractive because it requires discretisation only at the design boundary (at the zero level set contour). This reduction of problem dimensionality considerably simplifies the re-meshing task (especially in three-dimensions), which can be performed efficiently and robustly. Thus, its rapid and robust re-meshing and accurate boundary solutions make the boundary based body mapping method a natural choice for the solution of LS based shape and topology optimisation problems.

The boundary based body conforming approaches have been progressively improved over the years. However, the research methods presented in the early stages for compliance minimisation, e.g. [9,17] are based on the initial guessed design with pre-existing holes. Therefore, in the absence of a hole nucleation mechanism, the optimal designs obtained are highly dependent on the initial guessed designs. Ullah et al. [22] proposed an evolutionary optimisation approach based on the BEM and LSM with a stress based hole insertion mechanism. The optimal designs generated with their proposed optimisation method do not rely on initial guessed designs with pre-existing holes. The stress based hole insertion criterion is further investigated for a possible correlation with a topological derivative based hole insertion mechanism in [23]. The BEM and LSM based evolutionary optimisation method is further extended for the solution of three-dimensional problems in [24].

The topology optimisation method presented in [22,23] has been successfully implemented with the stress based sensitivities for shape optimisation, and both stress and topological derivative based sensitivities [25,26] have been used to carry out topological changes. This paper presents a novel methodology where the shape and topological derivatives are used to evaluate the associated sensitivities for compliance minimisation within the BEM and LSM framework. This approach is further equipped with the implementation of a bisectioning algorithm which effectively preserves the volume thereby strictly satisfying the volume constraint. Moreover, the proposed optimisation method does not rely on an initial guessed design with pre-existing holes. Instead the topological derivative based hole insertion criterion [23] used allows automatic hole nucleation and makes this new approach insensitive to the choice of initial guessed design. During the optimisation process NURBS [27] are fitted through the zero level set contours, which links an implicit geometry representation to its structural model. Additionally, this provides an optimal design in a standard CAD format and without intermediate material densities, which can be directly used in other design processes. The proposed method uses the two-dimensional version of the BEM analysis software Concept Analyst (CA) [28]. CA is capable to automatically discretise the NURBS based structural model.

The combination of the BEM (boundary based body mapping) and LSM requires a comprehensive investigation to effectively utilise their attractive properties in the field of structural optimisation. Using this as a milestone, this paper presents a detailed implementation of the use of BEM in a *sensitivity* and LSM based structural topology optimisation. Sections 2 and 3 present overviews of the shape sensitivity analysis and level set based structural optimisation, respectively. The BEM is developed in Section 4. We present details of the optimisation algorithm and its implementation in Section 5. The results obtained from the proposed algorithm are presented and discussed in Section 6. The paper closes with some concluding remarks in Section 7.

2. Shape sensitivity analysis

In structural optimisation different objective functions can be used to evaluate the performance of a given structure subject to constraints in the design variables. In this study the design objective function is to find the optimal topology of a structure with minimum compliance subject to a volume constraint. Consider a