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Improved non-singular method of fundamental solutions for two-dimensional isotropic elasticity problems with elastic/rigid inclusions or voids

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ABSTRACT

In this work, an Improved Non-singular Method of Fundamental Solutions (INMFS) is developed for the solution of two-dimensional linear elasticity problems. The source points and field points are collocated on the physical boundary, while the conventional MFS requires a troublesome fictitious boundary outside the physical domain. In INMFS, the desingularization is, for complying with the displacement boundary conditions, achieved by replacement of the concentrated point sources by distributed sources over circular discs around the singularity, and for complying with the traction boundary conditions by assuming the balance of the forces. This procedure is much more efficient than the previously proposed procedure that involves two reference solutions and at the same time enables INMFS for solving problems with internal voids and inclusions. The method has been assessed by comparison with MFS, analytical solutions and previous desingularization technique. The method is easy to code, accurate, efficient, and straightforwardly extendable to three dimensions.

1. Introduction

In recent years, there has been a strong development of mesh reduction methods in which polygon-like meshes are reduced or avoided [1–7]. The Method of Fundamental Solutions (MFS) (sometimes also called the F-Trefftz method, charge simulation method, or singularity method [8–10]) is a numerical technique that belongs to the class of methods generally called boundary methods. The other well-known representative of these methods is the boundary element method (BEM) [11,12]. Both methods are best applicable in situations where a fundamental solution of the partial differential equation in question is known. In such cases, the dimensionality of the discretisation is reduced. The BEM, for example, requires polygonisation of the boundary surfaces in general three-dimensional (3D) cases, and boundary curves in general 2D cases. This BEM approach requires the solution of complicated regular, weakly singular, strongly singular, and hyper-singular integrals over boundary segments which is usually quite a cumbersome and a non-trivial task [13]. The MFS [14] has certain advantages over BEM that stem mostly from the fact that only the "pointisation" of the boundary is needed, which completely avoids any integral evaluations, and makes no principal difference in coding between the 2D and the 3D cases. On the other hand, when a singular fundamental solution is involved, the MFS requires nodes that are positioned on an artificial boundary located outside the computational domain. The location of the artificial boundary represents the most serious problem of the MFS and has to be at the present dealt with heuristically [15] or by some optimization procedure [16,17] that requires substantial additional computing time. The expansion coefficients of the solution in MFS are determined so that the solution satisfies the boundary condition with the help of direct collocation [14,15], least squares approximation [18] or by an integral fit of the boundary data [19,20]. Moreover, it has certain advantages over BEM, e.g. no singularity and no boundary integrals are required. Both BEM and MFS are ideal candidates for solving isotropic and anisotropic linear elasticity problems [21,22], since the fundamental solution for this type of problems is known [23,24].

In the traditional MFS, the determination of the proper distance between the real boundary and the fictitious boundary is troublesome. In recent years, various efforts have been made, aiming to remove this drawback of the MFS, so that the source points can be placed on the real boundary directly. Young et al. [25,26] were the

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first to propose to place the source points at the boundary in the MFS. They introduce novel ways to determine the diagonal collocation matrix coefficients. The diagonal coefficients were determined directly for simple geometries or by using the results from the BEM, based on the fact that the MFS and the indirect boundary integral formulation are similar in nature. In their approach, information of the neighbouring points before and after each source point is needed, in order to form line segments for integrating the kernels to obtain the diagonal coefficients. This is essentially the same information of the element connectivity as in a BEM mesh. A subtracting and add-back technique was used to get rid of the artificial boundary in [27]. A similarly modified MFS was proposed in [28], where the diagonal terms are determined by the integration of the fundamental solution on the line segments formed by using neighbouring points, and the use of a constant solution to determine the diagonal coefficients of the derivatives of the fundamental solution in different coordinate directions. This approach is very stable, but it amounts to solve the problem twice. The group of Chen [29] improved the evaluation of the singular and nearly singular kernels in MFS for two-dimensional (2D) elasticity problems, based on the evaluation strategy, derived originally for BEM [13]. They also [30] proposed a non-singular MFS for determining the diagonal coefficients in the modified MFS by applying a known solution inside the domain, so that the diagonal coefficients from both the fundamental solution and its derivative can be determined indirectly, without using any element or integration concept. Again, this approach is appealing, stable, and accurate but it is costly for solving large-scale problems due to the need to solve the problem twice. The solution also depends on the choice of the internal reference points. The singular boundary method is applied to two-dimensional (2D) elasticity problems in [31], by using an inverse interpolation technique to regularize the singularity of the fundamental solution of the equation governing the problem of interest the regularized meshless method for the nonhomogeneous problems in conjunction with the dual reciprocity technique for the evaluation of the particular solution is given in [32]. The present authors [33,34] recently presented a new boundary meshfree approach named Non-singular MFS (NMFS) for isotropic and anisotropic elasticity problems based on the Boundary Distributed Source (BDS) method [35], which has been recently extended to solve the porous media problems with moving boundaries [36] and Stokes flow problems [37]. The NMFS has been developed to solve also the multi-body elastic problems [38] and 3D elasticity problems with displacement boundary conditions [39]. The NMFS has no fictitious boundaries and singularities. In NMFS, the concentrated point sources are replaced with area-distributed sources covering the source points for 2D problems. These area-distributed sources represent analytical integration of the original singular fundamental solution, so that they preserve the advantage of diagonal dominance for the system of equations, while they have no troublesome singularity issues. Liu [33] used the approach of Šarler [28] to determine the diagonal coefficients of the derivatives of the fundamental solution. The problem with the NMFS is that a careful selection is needed for reference solutions. Recently, Kim [40] suggested a much simpler way to determine the diagonal elements for the Neumann boundary conditions by invoking that the boundary integration of the normal gradient of the potential should vanish. In the present paper, the approach from [40] is extended from potential problems to linear elasticity problems. This approach can be applied also to the external domain problems, which previously could not be tackled by the NMFS. Numerical examples, relevant to micromechanics problems, with mixed boundary conditions are presented. The feasibility and the accuracy of the newly developed approach is demonstrated for problems of deformation of bodies with elastic or rigid inclusions and/or voids.

2. Governing equations

Consider a two-dimensional solid confined to domain Ω with boundary Γ. The solid behaviour is ideally isotropic elastic. Let us introduce a two-dimensional Cartesian coordinate system with orthonormal base vectors $\mathbf{i}_x$ and $\mathbf{i}_y$ and coordinates p_x and $p_y$of point P with position vector $\mathbf{p} = p_x\mathbf{i}_x + p_y\mathbf{i}_y$. The solid is governed by Navier's equations for plane strain problems, which are the conditions for equilibrium, expressed with the displacement $\mathbf{u}$

$$\frac{2(1-\nu)}{1-2\nu}\frac{\partial^2 u_x(\mathbf{p})}{\partial p_x^2} + \frac{\partial^2 u_x(\mathbf{p})}{\partial p_y^2} + \frac{1}{1-2\nu}\frac{\partial^2 u_y(\mathbf{p})}{\partial p_x \partial p_y} = 0, \tag{1}$$

$$\frac{2(1-\nu)}{1-2\nu}\frac{\partial^2 u_y(\mathbf{p})}{\partial p_y^2} + \frac{\partial^2 u_y(\mathbf{p})}{\partial p_x^2} + \frac{1}{1-2\nu}\frac{\partial^2 u_x(\mathbf{p})}{\partial p_x \partial p_y} = 0, \quad \mathbf{p} \in \Omega \cup \Gamma, \tag{2}$$

where ν is Poisson's ratio. The boundary is divided into two not necessarily connected parts $\Gamma = \Gamma^D + \Gamma^T$. On the part Γ^D the displacement (Dirichlet) boundary conditions are given, and on the part Γ^T the traction (Neumann) boundary conditions are given (see Fig. 1).

$$u_\varsigma(\mathbf{p}) = \bar{u}_\varsigma(\mathbf{p}); \qquad \varsigma = x, y, \quad \mathbf{p} \in \Gamma^D, \tag{3}$$

$$t_\varsigma(\mathbf{p}) = \bar{t}_\varsigma(\mathbf{p}); \qquad \varsigma = x, y, \quad \mathbf{p} \in \Gamma^T, \tag{4}$$

where $\bar{u}_\varsigma$ and $\bar{t}_\varsigma$ representing known functions. The strains $\varepsilon_{\varsigma\xi}$; $\varsigma, \xi = x, y$ are related to the displacement gradients by

$$\varepsilon_{\varsigma\xi} = \frac{1}{2}\left(\frac{\partial u_\varsigma}{\partial p_\xi} + \frac{\partial u_\xi}{\partial p_\varsigma}\right). \tag{5}$$

The stress components $\sigma_{\varsigma\xi}$; $\varsigma, \xi = x, y$ are for the plane strain cases related to the strains through Hooke's law

$$\sigma_{\varsigma\xi} = \lambda\delta_{\varsigma\xi}(\varepsilon_{xx} + \varepsilon_{yy}) + 2\mu\varepsilon_{\varsigma\xi}, \tag{6}$$

where $\mu = E/2(1+\nu)$ is the shear modulus of elasticity, E is modulus of elasticity, or Young's modulus, $\lambda = 2\nu\mu/(1-2\nu)$ is Lamé constant, and $\delta_{\varsigma\xi}$ is the Kronecker delta

$$\delta_{\varsigma\xi} = \begin{cases} 1, & \varsigma = \xi; \\ 0, & \varsigma \neq \xi. \end{cases} \tag{7}$$

The formulation for plane stress problems can be obtained by introducing the modified Poisson's coefficient ν' and modified Young's modulus E', defined as

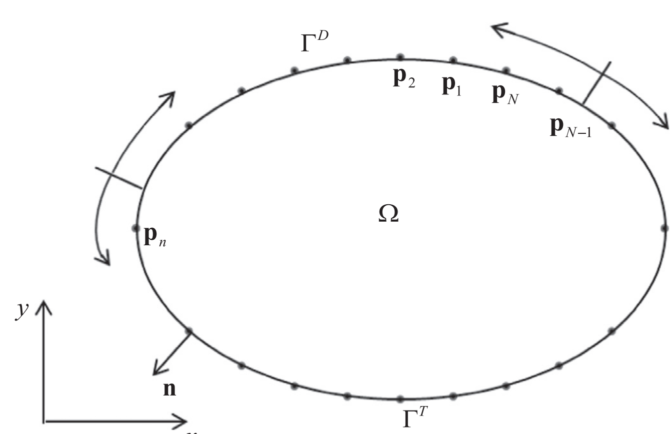

Fig. 1. Problem domain Ω with displacement (Dirichlet) Γ^D and traction (Neumann) Γ^T parts of the boundary.