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Simulation on the interaction between multiple bubbles and free surface with viscous effects

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ABSTRACT

Based on the boundary layer theory, a general Bernoulli equation involving normal and tangential stresses has been derived and the weak viscous effects have been considered. Three-dimensional boundary element method and Green's function have been adopted to solve the interaction between bubbles and free surface. Numerical results have been validated by convergence study and comparison with published results. On this basis, two in-phase and out-phase bubbles in the vicinity of free surface are chosen as cases at different Froude number. Influence of fluid viscosity or Reynolds number is mainly investigated. Physical relevance of numerical computation and the range of validity of numerical simulation are further discussed. It is found that viscous effects depress the interaction between bubbles and free surface, which hinders the formation of the downward jet on the upper bubble surface and declines bubble volume and the height of free-surface spike.

1. Introduction

Understanding of dynamic behaviours of multiple bubbles contained in the viscous fluid in the vicinity of free surface is of great significance in various engineering applications. Famous examples may include bubble damage on tissue and cells [5], underwater explosion near free surface [18,42], micro-bubble drag reduction [25], etc. From the viewpoint of mathematical modelling and numerical simulation, there are two challenging aspects distinguishing this problem. One is the strong nonlinear interaction between deformable bubbles and free surface and the other is the effects of the fluid viscosity on the interaction.

In terms of simulating interactions between bubbles and free surface, one of the most common methods is boundary element method (BEM) [1,3,40,32,36,9,41,12,13,44]. With assumptions of incompressibility of an ideal fluid and irrotationality of the flow, Wang et al. [34] adopted BEM to study the nonlinear interaction between a bubble and free surface for the axisymmetric case with buoyancy effect. Their numerical results were compared with the experimental observation of Blake and Gibson [2]. After the jet generated from one side of the bubble surface touched the other side, Wang et al. [35] introduced an axisymmetric vortex ring at the toroidal bubble stage to deal with the discontinuity of velocity potential over the joint surface after the jet impact. The potential was decomposed into the known discontinuous potential of a vortex ring and an unknown continuous potential. Zhang et al. [39] carried out computations of two bubbles beneath free surface for the three-dimensional (3D) case. A follow-on work by Zhang et al. [40] then extended the axisymmetric vortex ring model to the 3D problem. Robinson et al. [30] and Pearson et al. [28] further simulated the motion of one and a vertical column of two bubbles beneath free surface. Recently, Ni et al. [26,27] studied the bursting of a bubble at the free surface by using BEM. The initial pressure inside the bubble was equal to and higher than ambient pressure, respectively. In the former case, the bubble stayed at the free surface almost stationary and its upper surface virtually coincided with free surface; in the latter case, the bubble would rise to the free surface and burst open violently, without equilibrium phase. After the bubble was open, a jet would shoot up from the bottom of the bubble before the narrow tip split from the jet and form a string of droplets.

Based on the interaction between bubbles and free surface, effect of fluid viscosity was further studied. In order to include viscous effect, one direct method is to solve the Navier–Stokes equation by using computational fluid dynamics (CFD), which needs large calculation amount usually. An alternative method is 'boundary layer theory', which confines the viscous effects of fluid to the thin boundary layer around the boundary surface. Miksis et al. [23] introduced the boundary layer theory into bubble dynamics. They assumed there was a very thin boundary layer around the bubble, beyond which the fluid was inviscid and

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incompressible and the flow was irrotational. Thus a velocity potential theory could be used in the fluid domain except inside the boundary layer. They only included the normal viscous stress and ignored the velocity variation on both sides of the boundary layer. Lundgren and Mansour [22] analysed the boundary layer equation and obtained the differences of pressure and normal velocity on both sides of the boundary layer. Boulton-Stone and Blake [5] and Boulton-Stone [6] extended the method of Lundgren and Mansour [22] and made the tangential velocity on both sides of the boundary layer continuous. On this basis, they simulated the bursting of a bubble with ambient pressure inside at the free surface for an axisymmetric case. Georgescu et al. [10] also studied the bubble bursting at the free surface by using the axisymmetric BEM, where the normal viscous stress was included and an explicit second-order time progression scheme was adopted. Klaseboer et al. [16] adopted the idea of Joseph and Wang [14] to improve the bubble boundary layer theory in axisymmetric case. They considered the continuity of both normal and tangential stresses of a bubble and simulated the rising of the bubble at high Reynolds number. Lind and Phillips [20] introduced an extra stress tensor into the general Bernoulli equation on the bubble surface in axisymmetric case and derived this tensor for Newtonian fluid and viscoelastic fluid, respectively. The tensor for Newtonian fluid was just the normal viscous stress as that in Georgescu et al. [10] actually. The work on a bubble near a rigid wall in Lind and Phillips [20] was then extended to that near a free surface by Lind and Phillips [21]. Zhang and Ni [43] first extended the bubble boundary layer theory into 3D case. They simulated the evolution of a 3D bubble in unbounded fluid domain and found the jet velocity was depressed by the fluid viscosity.

Within the framework of BEM with boundary layer theory to include the weak viscous effect of fluid, it seems that there has been little work which considers the interaction between multiple bubbles and free surface in three dimensions. Although the case of two bubbles has been reported by Zhang and Ni [43], there has been lack of investigations on how the effect of free surface is involved and whether viscous effect will affect the desired results such as the spike of free surface and the jets of bubbles. This forms the prime motivation of the present work and gives its distinctive focus. Based on the 3D bubble model set up by Zhang and Ni [43], this paper extends the model to involve the effect of free surface. Convergence study with time step and mesh density is conducted and numerical results are compared with those in Lind and Phillips [21]. On this basis, case studies of the interaction between in-phase or out-phase bubbles and free surface are investigated. Influence of Reynolds number and physical relevance of numerical computation are further discussed. The proposed model and method are not only important from academic perspective, but also of significance in the practical application. For example, it has the potential to be used in the micro-bubble and air layer drag reduction [38], especially in the vicinity of free surface. The original BEM method which ignores the viscous effect of the fluid is quite limited for this kind of problem. In contrast, by combining BEM method and boundary layer theory, the proposed method in this paper has the advantage to include viscous effects and of high computation efficiency, which is suitable for this kind of practical problem.

2. Mathematical model and numerical method

2.1. Velocity potential flow and Green's function

Fig. 1 gives the sketch of the problem to be considered, which shows two bubbles with high internal pressure underneath the initially stationary free surface. We define a Cartesian system

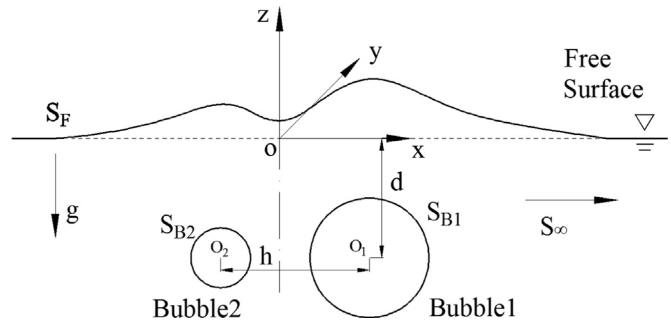

Fig. 1. Sketch of the problem with Cartesian coordinate system.

$O-xyz$ with the origin O at the undisturbed flat free surface and z axis pointing upwards. The initial submergence of the two bubble centres are equal and defined as d. The initial distance between the bubble centres is h.

Beyond the very thin boundary layer, it is assumed that the fluid is inviscid and incompressible and the flow is irrotational. Therefore, a velocity potential Φ can be introduced, which satisfies Laplace's equation

$$\nabla^2\Phi = 0, \tag{1}$$

in the fluid domain out of the boundary layer. By using the Green's third identity with the Green function G, this partial differential equation can be converted into the following boundary integral equation:

$$\tilde{\varepsilon}(\boldsymbol{p})\Phi(\boldsymbol{p}) = \iint_S \left(\frac{\partial \Phi(\boldsymbol{q})}{\partial n_q} G(\boldsymbol{p},\boldsymbol{q}) - \Phi(\boldsymbol{q}) \frac{\partial}{\partial n_q} G(\boldsymbol{p},\boldsymbol{q}) \right) dS, \tag{2}$$

where S is the domain boundary including the free surface S_F, the bubble surfaces S_B and the boundary S_∞ at infinity. In the equation $\boldsymbol{p}$ is the field point and $\boldsymbol{q}$ is the integral (or the source) point on the boundary. $\tilde{\varepsilon}(\boldsymbol{p})$ is the ratio between the solid angle when observing the flow field at point $\boldsymbol{p}$ and 4π. $\tilde{\varepsilon}(\boldsymbol{p}) = 1$ when $\boldsymbol{p}$ lies inside the fluid domain and $\tilde{\varepsilon}(\boldsymbol{p}) = 0.5$ when $\boldsymbol{p}$ lies on the smooth boundary. The fundamental solution for 3D potential problems is $G(\boldsymbol{p},\boldsymbol{q}) = \frac{1}{4\pi|\boldsymbol{R}-\boldsymbol{r}|}$, where $\boldsymbol{R} = (X, Y, Z)$ and $\boldsymbol{r} = (x, y, z)$ are the position vectors of $\boldsymbol{p}$ and $\boldsymbol{q}$, respectively. Because of the boundary condition at infinity $\nabla\Phi \to 0$, where the fluid is assumed to be undisturbed, the integration over S_∞ can be replaced with that over a control surface S_C at a large distance from the body, where $\Phi = 0$ is used.

The fully-nonlinear kinematic boundary condition on both bubble and free surfaces can be written as below in Lagrangian framework:

$$\frac{D\boldsymbol{x}}{Dt} = \nabla\Phi. \tag{3}$$

where $\boldsymbol{x}$ is the position vector of the node on the bubble or free surface and t is time.

2.2. Boundary layer theory and the general Bernoulli equation

In the very thin boundary layer, for an incompressible Newtonian fluid the velocity $\boldsymbol{U}$ obeys the Navier–Stokes equation as below:

$$\frac{\partial \boldsymbol{U}}{\partial t} + (\boldsymbol{U}\cdot\nabla)\boldsymbol{U} = \boldsymbol{g} - \frac{\nabla P}{\rho} + \frac{\mu}{\rho}\nabla^2\boldsymbol{U}, \tag{4}$$

where $\boldsymbol{g}$ is acceleration due to gravity, P is pressure, ρ is the density of the fluid, and μ is dynamic viscous coefficient of the fluid.

Based on the Helmholtz decomposition, the velocity $\boldsymbol{U}$ can be