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Nonlinear simulation of resonant sloshing in wedged tanks using boundary element method

Chongwei Zhang

Department of Mechanical Engineering, University College London, Torrington Place, London WC1E 7JE, UK

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ABSTRACT

Nonlinear sloshing problems in wedged tanks are simulated based on the paralleled boundary element method. An improved semi-Lagrangian procedure is introduced for the free-surface updating near non-vertical tank walls. Two types of resonances are mainly considered. One is the classical resonance, which is induced by a lateral excitation on the tank. Featured sloshing phenomena in wedged tanks are observed compared with those in rectangular tanks. The other type is the Faraday waves due to the forced tank oscillation perpendicular to the undisturbed free-surface plane. Effects of the excitation frequency, acceleration amplitude, instability regions and initial free-surface disturbance on behaviours of the resonant sloshing in wedged tanks are investigated.

1. Introduction

The liquid sloshing, which means the free-surface motion in its container, is an important problem for liquefied natural gas (LNG) carriers. Associated with the sloshing phenomenon, the resonance with high nonlinearities is of major concern. When the resonant sloshing occurs, the induced violent waves might endanger the vessel stability and tank structures. At present, the nonlinear sloshing in wall-sided (e.g rectangular or cylindrical) tanks has been widely studied based on the weakly-nonlinear multimodal method [1–3], weakly-nonlinear perturbation expansion [4–7], shallow-water theory [8–9], pseudo-spectral method [10–12], boundary element method (BEM) [13–17], finite element method (FEM) [18–22], finite difference method (FDM) [23–27], finite volume method (FVM) [28], particle method [29–31] and mechanical analogy method [32–34].

Unlike the extensive investigations for wall-sided tanks as shown above, sloshing problems in non-wall-sided tanks have not stepped into the centre of the spotlight. Among the existing literatures, a primary group focuses on the natural sloshing frequencies and modes. For example, McIver [35] studied the natural frequencies for 2D circular and 3D spherical containers, Lukovsky and Timokha [36] developed an analytically-oriented modal approach for natural modes in the circular conical tank, Damattya et al. [37] conducted experiments for fundamental natural sloshing frequencies in the conical tank, and Gavrilyuk et al. [38] further extended the modal approach to solve natural frequencies in truncated conical tanks.

Another group of the studies concentrates on the sloshing behaviour. For example, Mitra et al. [40] concerned the linear sloshing in 2D trapezoidal and horizontal circular cylindrical containers using FEM. Hasheminejad and Mostafa Aghabeigi [41] developed a semi-analytical linear model for the half–full elliptical container excited laterally. Ramaswamy [42] studied the oscillation of viscous liquid in non-wall-sided tanks with varying depth, based on an arbitrary-Lagrangian–Eulerian (ALE) method. Behr and Abraham [43] simulated the viscous liquid sloshing in a trapezoidal tank based on FEM. Gavrilyuk et al. [44] and Gavrilyuk et al. [45] derived weakly nonlinear modal equations to describe the liquid sloshing in the truncated conical tank. Dai and Xu [46] developed a potential-flow FDM to simulate the sloshing in 3D horizontal cylindrical containers. Modaressi-Tehrani [47] modelled the transient liquid motion in a horizontal cylindrical tank with baffles using the FLUENT software. Zhou et al. [48] extended the techniques developed by Behr and Abraham [43] for free-surface motions in 2D circular tanks. Marsh et al. [49] simulated the liquid motion in liquid sloshing dampers of different geometries using the particle method.

It is found that the 3D wedged tank has rarely been considered in above literatures. However, the tank with wedged geometry could also have practical applications [39]. Thus, in Zhang [50], we investigated the nonlinear sloshing problem in 3D wedged tanks based on BEM. In particular, an improved semi-Lagrangian procedure (ISL) was introduced for the free-surface updating near

E-mail address: chongwei.zhang.11@ucl.ac.uk

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non-vertical walls. However, the resonant sloshing in the wedged tank was not discussed, due to lack of awareness about the resonance conditions (i.e. natural sloshing frequencies). Then, in Zhang [39], we further developed a numerical approach to predict the resonance conditions of a general liquid tank. Natural frequencies and modes for the 3D wedged tanks are systematically investigated. Eventually in the present study, we could achieve the nonlinear simulation of the resonant sloshing in wedged tanks by exciting the tank at the resonance condition. The numerical scheme based on the BEM is adopted for the simulation. The BEM requires the spatial discretization only on the boundary surface of the domain, which is distinctive from the domain methods that require a discretization of the entire region of the flow field. This saves the trouble of generating interior mesh elements for an irregular 3D fluid domain of the concerned sloshing problem. This feature also considerably reduces the number of unknowns that must be found at each time step, leading to a much smaller system of equations than those arising from domain methods. Although the resulting system of equations is normally dense, the concerned problem only requires a few thousands of mesh elements, so that the simulations could run on personal computers without much challenge of computing resources.

The content of the article is organized as follows. In Section 2, the mathematical model of the nonlinear sloshing problem would be given based on the potential-flow theory. The ISL procedure used to update the free surface is described. In Section 3, numerical details of the paralleled boundary element method would be presented. Then, the resonant sloshing in wedged tanks is studied in Section 4. Two types of resonances are mainly considered. The first type is the classical resonance, which occurs due to a lateral tank oscillation at certain resonance frequencies. The second type is the Faraday waves, which is observed in the unstable status when the tank is excited perpendicular to the plane of an undisturbed free surface. Conclusions are drawn in Section 5.

2. Mathematical equations

To describe the sloshing problem, two coordinate systems are defined. One is the earth-fixed coordinate system $O_o - x_o y_o z_o$ with $O_o z_o$ pointing vertically upward. The other is the tank-fixed coordinate system $O - xyz$ with O located at the rotation centre of the tank. Initially, these two systems coincide with each other, and the still free surface is at $z = z_f$. The fluid domain, wetted surface of the tank and free surface are denoted by V, S_B and S_F, respectively. A scalar velocity potential $\varphi(x, y, z, t)$ whose gradient represents the fluid velocity is introduced. At any instant, the velocity potential φ could be determined from the following initial-boundary value problem expressed in $O - xyz$

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0, \text{ in } V, \tag{1}$$

$$\frac{\partial\varphi}{\partial n} = (\mathbf{v}_c + \mathbf{\Omega} \times \mathbf{r})\cdot\mathbf{n}, \text{ on } S_B, \tag{2}$$

$$\frac{\partial Z}{\partial t} - (\mathbf{v}_c + \mathbf{\Omega} \times \mathbf{r})\cdot\nabla Z + \nabla\varphi\cdot\nabla Z = 0, \text{ on } S_F, \tag{3}$$

$$\frac{\partial\varphi}{\partial t} - (\mathbf{v}_c + \mathbf{\Omega} \times \mathbf{r})\cdot\nabla\varphi + \frac{1}{2}\nabla\varphi\cdot\nabla\varphi + gz_o = 0, \text{ on } S_F, \tag{4}$$

$$\varphi = 0 \text{ and } \partial\varphi/\partial t = 0, \textit{ for } t = 0 \tag{5}$$

where $\mathbf{v}_c = \{v_1, v_2, v_3\}$ is the velocity of the origin O, $\mathbf{\Omega}$ is the angular velocity of the $O - xyz$ system about O, $\mathbf{r}$ denotes the position vector, $\mathbf{n}$ is the unit normal vector pointing out of the fluid domain, and $Z(x, y, z, t) = 0$ is an implicit definition of the free-surface profile. The variable under $O_o - x_o y_o z_o$ is expressed with the subscript o. Eq. (3) is the kinematic free-surface boundary condition, which assumes that a fluid particle on the free surface will always stay on the free surface. Eq. (4) is the dynamic free-surface boundary condition, which is obtained by setting the pressure at the free surface to be constant. Both free-surface boundary conditions are satisfied on the instantaneous free surface.

Note that, although the Laplace's equation of Eq. (1) is a linear partial differential equation, the nonlinear free-surface boundary conditions have made the sloshing problem nonlinear. Two steps are required to make this nonlinear problem solvable with BEM. In the first step, the time-stepping is performed on the kinematic free-surface boundary condition to predict the free-surface position and the dynamic one to calculate the velocity potential on it. In the second step, the Laplace's equation of the velocity potential could be solved by BEM, with the velocity potential and its normal derivative known on the free surface and wetted tank surface, respectively.

The ISL procedure [50] is used for the time-stepping of free-surface boundary conditions. The essential idea of this procedure is illustrated in Fig. 1(a). A path is artificially defined through each node of the free-surface mesh. The mesh nodes could slide along the paths, reflecting the deformation of the free surface mesh. On the water line, we set 'body-fitted' paths, so that the waterline

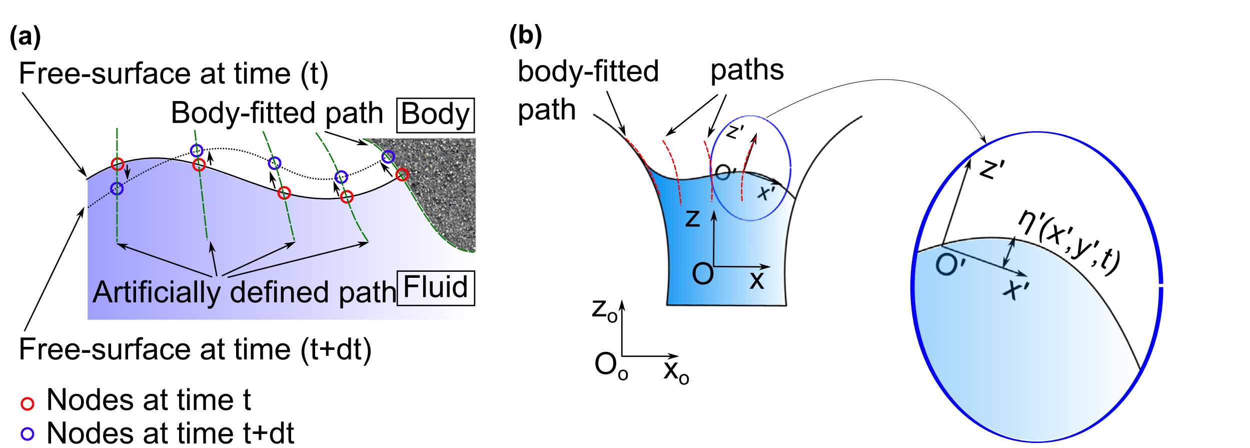

Fig. 1. Sketch of improved semi-Lagrangian free-surface updating procedure: (a) idea description; (b) local coordinate system on free surface.