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Meshfree approach for linear and nonlinear analysis of sandwich plates: A critical review of twenty plate theories

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ABSTRACT

Present paper reviews twenty different theories used for analysis of multilayered plates. The mathematical formulation of the actual physical problem of the plate subjected to mechanical loading is presented using von Karman nonlinear kinematics. These non-linear governing differential equations of equilibrium are linearized using quadratic extrapolation technique. A meshfree approach based on polynomial radial basis function is used for obtaining the solution. The results obtained for the sandwich plate are validated with other available results. It is observed that some theories under predicts the deflection by a reasonable amount. The effect varying core thickness on stresses and deflection of the sandwich plate is also presented.

1. Introduction

Sandwich structures have long been recognized as one of the weight efficient plate construction. In a situation where weight minimization is a major concern, particularly in aerospace industry, sandwich constructions having core sheets with light weight and low strength and face sheets with high strength provides the solution. The sandwich structures result in substantially high strength and stiffness of the structure with reasonably low weight. The sandwich structures are also being used in engineering applications where energy absorption due to impact loading is a major concern. Mantari et al. [1] presented a solution for laminated composite and sandwich plates using trigonometric shear deformation theory. Analytical solutions for static analysis of sandwich plates using Navier's method are presented by Kheirikhah et al. [2]. Ferreira [3] presented a poly-harmonic thin plate radial basis function for the bending response of layer-wise modeled laminated and sandwich plates. Ferreira et al. [4] used multiquadric radial basis function method and trigonometric shear deformation theory for bending analysis of isotropic, laminated and sandwich plate. Castro et al. [5] used wavelet collocation and layer-wise theory for the analysis of laminated composite and sandwich plates. Xiang et al. [6] presented static response of isotropic, laminated and sandwich plates using the inverse multiquadric radial basis function. The plates were modeled using four different shear deformation theories. Husain et al. [7] have studied the large deflection of thin plates with immovable edges using polynomial radial basis function based meshless method. Bitaraf and Mohammadi [8] have used a finite point approach for solving the nonlinear partial differential equations of fourth order to analyze large deflection behavior of plates. Some of the displacement fields used for analysis of plates by researchers are taken for present analysis and are shown in Table 1 [9–33].

2. Mathematical formulation

The displacement field and governing differential equations of plate at any point on the plate made up of perfectly bonded layers of uniform thickness is expressed as Singh and Shukla [34]:

$$\begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} u_0(x,y) - z\dfrac{\partial w_0(x,y)}{\partial x} + f(z)\psi_x(x,y) \\ v_0(x,y) - z\dfrac{\partial w_0(x,y)}{\partial y} + f(z)\psi_y(x,y) \\ w_0(x,y) \end{Bmatrix} \quad (1)$$

The parameters U, V and W are the in-plane and transverse displacements in plate at any point (x, y, z) in x, y and z direction respectively. Variables u_0, v_0 and w_0 are the displacements at mid plane of the plate at any point (x, y) in x, y and z direction respectively. The variables ψ_x and ψ_y are the rotations of the normal

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Table 1
Transverse shear function $f(z)$.

Notation	$f(z)$	Proposed/Used by
T1	$z.\left[1-\frac{4}{3h^2}z^2\right]$	Levinsion [9], Reddy [10]
T2	$z.\left[\frac{1}{4}-\frac{5}{3h^2}z^2\right]$	Shimpi and Patel [11]
T3	$\frac{z}{2}.\left[\frac{h^2}{4}-\frac{z^3}{3}\right]$	Ambartsumian [12]
T4	$z.\left[\frac{5}{4}-\frac{5}{3h^2}z^2\right]$	Kruszewski [13], Panc [14], Reissner [15]
T5	$\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$	Levy [16], Stein [17], Touratier [18] Ghugal and Sayyad [19]
T6	$\sin\left(\frac{\pi z}{h}\right)$	Shimpi et al. [20] Ferreira et al. [21]
T7	$\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)-z$	Thai and Vo [22]
T8	$\tan(m.z)-m.z\sec^2\left(\frac{m.h}{2}\right),\ m=\left(\frac{1}{5h}\right)$	Mantari et al. [1]
T9	$z.m^{(-2(z/h)^2},\ m=3$	Mantari et al. [23]
T10	$z.e^{(-2(z/h)^2}$	Karama et al. [24]
T11	$z.m^{(-2(z/h)^2/\log m},\ m=3$	Aydogdu [25]
T12	$z.\cosh\left(\frac{1}{2}\right)-h.\sinh\left(\frac{z}{h}\right)$	Soldatos [26]
T13	$h.\sinh\left(\frac{z}{h}\right)-z.\cosh\left(\frac{1}{2}\right)-z$	Bessaim et al. [27]
T14	$\mathrm{asinh}\left(r\frac{z}{h}\right)-\left(\frac{2.z.r}{h\sqrt{r^2+4}}\right),\ r=0.3$	Grover et al. [28]
T15	$\frac{3\pi}{2}\left(h.\tanh\left(\frac{z}{h}\right)-z.\sec^2\left(\frac{h}{2}\right)\right)$	Akavci [29]
T16	$\frac{3\pi}{2}\left(h.\tanh\left(\frac{z}{h}\right)-z.\sec^2\left(\frac{1}{2}\right)\right)-z$	Daouadji [30]
T17	$h.\sinh\left(\frac{z}{h}\right)-\frac{4z^3}{3h^2}\cosh\left(\frac{1}{2}\right)$	Zenkour [31]
T18	$-z\left(\mathrm{sech}\left(\frac{\pi}{4}\right)\left(1-\frac{\pi}{2}\tanh\left(\frac{\pi}{4}\right)\right)+\left(1-\mathrm{sech}\left(\frac{\pi z^2}{h^2}\right)\right)\right)$	Daouadji et al. [32]
T19	$z.\mathrm{sech}\left(\frac{\pi z^2}{h^2}\right)-z.\ \mathrm{sech}\left(\frac{\pi}{4}\right)\left(1-\frac{\pi}{2}\tanh\left(\frac{\pi}{4}\right)\right)$	Akavci [29]
T20	$\sinh\left(\frac{z}{h}\right)e^{m.\cos\left(\frac{\pi z}{h}\right)}+m.\left(\frac{\pi z}{h}\right),\ m=0.5$	Mantari et al. [33]

to the mid plane due to shear deformation about x and y axes, respectively.

The governing differential equations of the rectangular plate are derived using the Hamilton's principle, which is expressed as:

$$\begin{aligned}
&\delta u_0:\ \frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\
&\delta v_0:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=0\\
&\delta w_0:\ \left(\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}\right)\left\{\frac{\partial w_0}{\partial x}\right\}+\left(\frac{\partial N_{yy}}{\partial y}+\frac{\partial N_{xy}}{\partial x}\right)\left\{\frac{\partial w_0}{\partial y}\right\}+N_{xx}\frac{\partial^2 w_0}{\partial x^2}\\
&\qquad +N_{yy}\frac{\partial^2 w_0}{\partial y^2}+2N_{xy}\frac{\partial^2 w_0}{\partial x\partial y}+\frac{\partial^2 M_{xx}}{\partial x^2}\\
&\qquad +\frac{\partial^2 M_{yy}}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}-q_z=0\\
&\delta\psi_x:\ \frac{\partial M^f_{xx}}{\partial x}+\frac{\partial M^f_{xy}}{\partial y}-Q^f_x=0\\
&\delta\psi_x:\ \frac{\partial M^f_{xy}}{\partial x}+\frac{\partial M^f_{yy}}{\partial y}-Q^f_y=0
\end{aligned}\tag{2}$$

The boundary conditions for simply supported edges are:

$$\begin{aligned}
&x=0,a:\ v_0=0;\ \psi_y=0;\ w_0=0;\ M_{xx}=0;\ N_{xx}=0\\
&y=0,b:\ u_0=0;\ \psi_x=0;\ w_0=0;\ M_{yy}=0;\ N_{yy}=0
\end{aligned}\tag{3}$$

3. Solution methodology

The method of total linearization through quadratic extrapolation is used as Singh and Shukla [34]. Meshfree methods were born with an objective of eliminating part of the difficulties associated with reliance on a mesh to construct the approximation in finite element method. In meshfree methods, the approximations are built from nodes only Governing differential Eq. (2) and boundary conditions (3) are discretized using polynomial radial basis function (RBF). RBF based meshfree formulation works on the principle of interpolation of scattered data over entire domain as shown in Fig. 1. Polynomial RBF is expressed as Naffa et al. [35]:

$$g=r^m \tag{4}$$

Where, Where, $r=\|X-X_j\|=\sqrt{(x-x_j)^2+(y-y_j)^2}$ and 'm' is shape parameter. In present analysis value of m=5 is taken after validation and convergence study. The theory of meshless method is detailed in Annexure-1.

The field variables (displacements) in terms of RBF are expressed as:

$$\begin{aligned}
u_0&=\sum_{j=1}^{N}\alpha_j^{u_0}g(\|X-X_j\|),\ v_0=\sum_{j=1}^{N}\alpha_j^{v_0}g(\|X-X_j\|),\ w_0\\
&=\sum_{j=1}^{N}\alpha_j^{w_0}g(\|X-X_j\|),\ \psi_x=\sum_{j=1}^{N}\alpha_j^{\psi_x}g(\|X-X_j\|),\ \psi_y\\
&=\sum_{j=1}^{N}\alpha_j^{\psi_y}g(\|X-X_j\|)
\end{aligned}\tag{5}$$

The governing differential Eq. (2) are discretized and finally expressed in compact matrix form as: