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Element-subdivision method for evaluation of singular integrals over narrow strip boundary elements of super thin and slender structures

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ABSTRACT

In this paper, based on the numerical investigation of singular integrals over narrow strip boundary elements stemming from BEM analysis of thin and slender structures with different numbers of Gauss points, an efficient method is proposed for evaluating the narrow strip singular boundary integrals using an adaptive unequal interval element-subdivision method in the intrinsic parameter plane. In this method, the size of the sub-element closest to the singular point is determined first in terms of the orders of the shape functions along two intrinsic coordinate directions. Then, the sizes of other sub-elements are computed by employing a criterion proposed by Gao and Davies [1,2] for evaluating nearly singular integrals in terms of an allowed number of Gauss points and the distance from the source point to the sub-element. The features of the proposed method are that the computational accuracy of various orders of singular integrals is controlled by the upper bound of the error of Gauss quadrature, rather than through artificially giving the size of the sub-elements and number of Gauss points, and because of using the unequal interval element-subdivision method, the number of required sub-elements is not large even for an element with high aspect ratio, usually less than 10 for a plate with aspect ratio of 100:1. A number of numerical examples for plates and shells with different aspect ratios are analyzed for various orders of integrals to demonstrate the efficiency of the proposed method.

1. Introduction

Thin-walled structures, such as plates, shells, and coatings, are commonly used structural members in engineering. For these structures, because the thickness is much smaller than the transversal size, it is a challenging task to carry out the thermal and mechanical analysis of such structures using a numerical method. When using the finite element method (FEM) to analyze this type of problems, usually there are two discretization schemes. One is to use brick elements to accurately model the thin walled structures. The fatal problem occurring in this scheme is that huge number of elements may be required because the element size should be in the same order as the thickness of the plate or shell. The second scheme, which is frequently used in FEM, is to use simplified element types to model the thin structure, for example, to use the plate or shell elements [3–6]. The drawback of this scheme is that the interaction mechanism between the plate and the out-plane media cannot be accurately reflected, and this may limit the use of FEM to solve composite structural problems with thin-walled components.

The boundary element method (BEM) has distinct advantage over FEM in the analysis of thin-walled structural problems, since only the surface of the structure needs to be discretized into boundary elements and no simplifying assumptions are imposed [7]. In addition, the basic physical quantities obtained using BEM are displacements and tractions on the surface. In view of the fact that the normal and tangential tractions on the surface can directly reflect the strength of the tensile and shear forces, it can be seen that BEM is very effective to perform the failure analysis of coating-like structures. However, when using BEM to solve thin-plate-like problems, the boundary element discretization according to the size of the surface usually results in the presence of narrow strip elements with high aspect ratio on the flanks of the plate [7–9]. As analyzed in Section 3, to achieve a satisfactory integration result over these elements, the required number of Gauss points exceeds far what is provided in a conventional BEM code [2]. This is why many researchers often obtain incorrect

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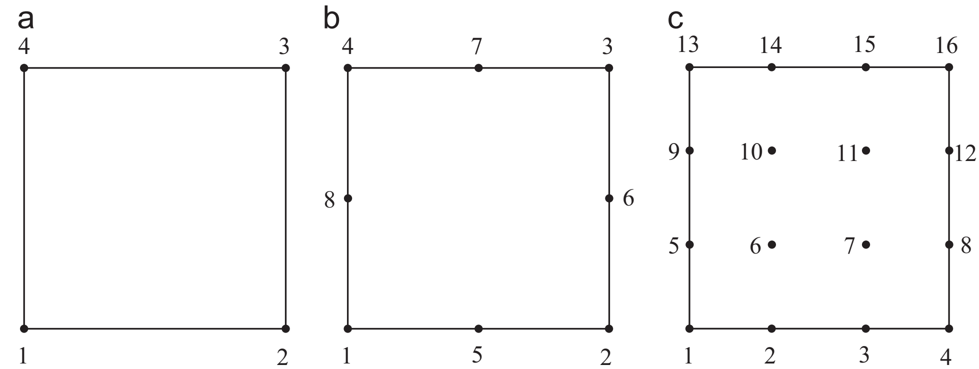

Fig. 1. Frequently used surface elements: (a) 4-noded linear; (b) 8-noded quadratic; (c) 16-noded cubic elements.

computational results when performing elasticity analysis of very thin plates using BEM. Therefore, in the simulation of thin-walled structures using BEM, apart from the need of treating nearly singular integrals [9–11] when an element under integration is located on the surface opposite the source point, a special integration scheme is also needed for treating the singular integrals over the flank elements when the source point is located on these elements.

For the evaluation of the singular and nearly singular integrals over surfaces of thin-walled structures, Sladek and Tanaka presented a regularization approach in [12] and gave a comprehensive review in [13]. Regarding the issue of evaluating the singular integrals over the narrow strip boundary elements, we made a detailed numerical investigation for various aspect ratio cases using different numbers of Gauss points and found that to achieve an acceptable integration accuracy, many Gauss points are required along the long intrinsic coordinate direction of the element, usually more than 50 (the details will be reported in Section 3 of this paper). However, when developing a BEM code, a limited number of Gauss points is allowed, usually less than 10. This is why people cannot obtain correct results when computing a super thin plate using the conventional BEM codes developed based on the treatment of singular integrals over regular aspect ratio elements by triangulation technique [2] or other direct methods [14–17]. Unfortunately, very few people realized this phenomenon and paid attention to research this problem. This paper is an attempt to study this issue deeply. Firstly, a numerical investigation is performed in Section 3 about the computational accuracy of various orders of singular boundary integrals with respect to different aspect ratio elements, in the interest of finding problems causing the incorrect results using conventional BEM. Then, an element-subdivision method is proposed in Section 4 based on a criterion proposed by Gao and Davies [1,2] for evaluating nearly singular integrals in terms of an allowed number of Gauss points and the distance from the source point to the sub-element. In the proposed method, two new techniques are presented for the first time to determine the sizes of sub-elements. Some numerical examples for thin plates and shells are analyzed in Section 5 to demonstrate the efficiency of the proposed method.

2. Singular boundary integrals in BEM analysis

In three-dimensional elasticity problems, the boundary integral equations can be expressed as [2]

$$cu_i(P) = \int_\Gamma U_{ij}(P,Q)t_j(Q)d\Gamma(Q) - \int_\Gamma T_{ij}(P,Q)u_j(Q)d\Gamma(Q) \tag{1}$$

where $c = 1/2$ for smooth boundary points and $c = 1$ for interior points; P and Q represent the source and field points, respectively; u_j and t_j are displacements and tractions defined on the boundary Γ;

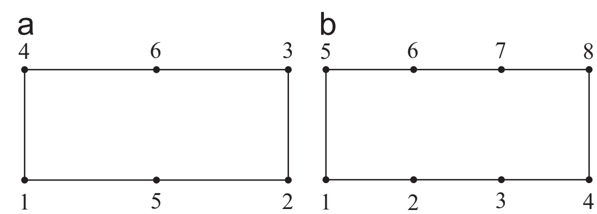

Fig. 2. Flank elements: (a) 6-noded element; (b) 8-noded element.

U_{ij} and T_{ij} are the Kelvin's fundamental solutions for displacements and tractions, which can be expressed as

$$U_{ij} = \frac{1}{16\pi(1-\nu)\mu\, r}[(3-4\nu)\delta_{ij} + r_{,i}r_{,j}] \tag{2}$$

$$T_{ij} = \frac{-1}{8\pi(1-\nu)r^2}\left\{\frac{\partial r}{\partial n}[(1-2\nu)\delta_{ij} + 3r_{,i}r_{,j}] + (1-2\nu)(n_i r_{,j} - n_j r_{,i})\right\} \tag{3}$$

in which, μ is the shear modulus, ν is Poisson's ratio, r is the distance between the source point and field point.

To evaluate the boundary integrals included in Eq. (1), the boundary Γ of the problem is discretized into a series of boundary elements [2] and integrations are performed over these elements. The discretized form of Eq. (1) can be written as

$$cu_i(P) = \sum_{e=1}^{ne}\left\{\sum_{\alpha=1}^{m} t_j^\alpha(Q^\alpha)\int_{S_e} U_{ij}(P,Q)N_\alpha(Q)d\Gamma(Q)\right\} - \sum_{e=1}^{ne}\left\{\sum_{\alpha=1}^{m} u_j^\alpha(Q^\alpha)\int_{S_e} T_{ij}(P,Q)N_\alpha(Q)d\Gamma(Q)\right\} \tag{4}$$

in which, ne is the number of elements, m is the number of element nodes, N_α is the shape function of the α-th element node.

Fig. 1 shows three types of frequently used surface boundary elements, i.e., 4-noded linear, 8-noded quadratic and 16-noded cubic elements. To match the 8-noded and 16-noded surface elements, two new elements, i.e. 6-noded and 8-noded elements, are derived for the flanks of a thin plate based on the Lagrange interpolation formulation, which are shown in Fig. 2. The shape functions for the first two types of elements in Fig. 1 can be found in [2] and those for remaining elements are listed in the Appendix of this paper. Our computational experience shows that in the analysis of a thin plate, the upper and lower surfaces are discretized using the 8-noded or 16-noded elements, while the flanks are narrow strip shaped and discretized using 6-noded or 8-noded strip elements as shown in Fig. 2. The 4-noded element as shown in Fig. 1(a) cannot achieve a stable result.

For the case that the source point is not located in the element under integration, standard Gaussian quadrature can be used to evaluate the boundary integrals in Eq. (1). However, when the source point is located on the element under integration, the