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Improving accuracy and efficiency of stress analysis using scaled boundary finite elements

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ABSTRACT

The scaled boundary finite element method (SBFEM) is a fundamental-solution-less boundary element method, which leads to semi-analytical solutions for stress fields. As only the boundary is discretized, the spatial dimension is reduced by one. In this paper, the SBFEM based polygon elements are utilized to improve the accuracy and efficiency of stress analysis. It retains the attractive feature of the SBFEM in solving problems with unbounded media and singularities. In addition, polygon elements are more flexible in meshing and mesh transition. Various measures which help improving accuracy or efficiency of the stress analysis, i.e. refining polygon mesh, nodal enrichment, appropriate placing of the scaling center, merging polygon elements and NURBS enhanced curved boundaries are discussed and compared. As a result, further insight into the refinement and improvement strategies for stress analysis is provided.

1. Introduction

As a versatile and robust numerical analysis method with powerful capability for simulating a large variety of problems with complex structural geometrics, complicated material properties, the finite element method (FEM) has found widespread application in engineering practice. However, the standard FEM yields relatively poor results when applied to problems containing curved boundaries, irregular inclusions or openings, problems associated with singular nature of the solution, such as reentrant corners and problems with unbounded domain subjected to forces which are not self-equilibrating. Besides, it may be time consuming in generating high quality meshes and performing mesh refinement to fit the exact geometry. In order to circumvent these difficulties, numerous methods have been developed in the literature, such as:

1. Improving the accuracy of geometric fitting by introducing p-version of FEM [1] and hp finite element [2]. As illustrated in [3], a meaningful high-order accurate solution in the presence of curved boundaries can only be obtained if the corresponding high-order approximation of the geometry is employed.
2. The use of polygon based hybrid stress approach [4], hybrid Trefftz formulation [5] to solve problems containing irregular inclusion or microstructure. It allows high flexibility in meshing and mesh transition.
3. Extended finite element which is suitable for modeling crack growth with minimal work of remeshing. However, it needs additional enrichment functions [6], which may compensate the advantages of reducing the computational effort.

In the late 1990s, a novel semi-analytical approach, the scaled boundary finite-element method (SBFEM) was proposed by Wolf and Song [7]. The scaled boundary finite-element method is a fundamental-solution-less boundary element method [8]. Only the boundary of the domain needs to be discretized resulting in the reduction of the spatial dimension by one. Moreover, singular stress field at the crack tip and bi-material interfaces are expressed semi-analytically without further adaptive refinement of the mesh. For the problem of complex geometry and complicated material properties, the domain are discretized into polygonal elements conveniently through Delaunay triangulation or Voronoi diagrams. Hence, it is widely applied to fracture mechanics [9–11]. As to the unbounded domain problems, SBFEM has been successfully applied to dynamic fluid-structure interaction [12] and soil-structure interaction analyses [13,14]. In addition, it also achieved great success in the solution of liquid sloshing [15], heat transfer problems [16] and electromagnetic problems [17].

There are numerous strategies to improve accuracy and efficiency of the stress analysis in the framework of SBFEM. Refinement of polygon mesh [18], h-hierarchical adaptive procedure [19] and appropriate placing scaling center to express stress singularity

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[20] are the most commonly used strategies. Moreover, the integration of SBFEM and the non-uniform rational B-Splines (NURBS) is a promising alternative. Employing NURBS as basis functions to construct exact geometric model can be traced back to isogeometric analysis (IGA) presented by Hughes [21]. In the subsequent field variable analysis, the basis is refined and/or its order elevated, while exact geometry is maintained at all levels of refinement. The IGA has motivated the development of a novel numerical analysis method and has successfully applied to various fields, such as solid mechanics, fluid mechanics and fluid-structure interaction, etc [22]. The SBIGA exploits the advantages of SBFEM and the IGA [23], higher convergence rate and higher efficiency are achieved in comparison with the standard SBFEM. In [24] SBIGA is applied to carry out time-domain dynamic analysis of dam-reservoir-foundation system taking into consideration the effect of water compressibility, the reservoir bottom absorption and soil-structure interaction of the unbounded foundation. In [25], the SBFEM enhanced IGA is further extended to deal with the problems of linear elastic fracture mechanics. It is also worth to note, in [26,27] NURBS enhanced finite element method (NEFEM) has been developed, which allows integration of the NURBS boundary representation of the domain and the FEM. However, for practical engineering implementation, inhomogeneity of the domain are often encountered, such as internal inclusions, gaps or openings, and various material properties in different parts of the domain, sophisticated treatment, e.g., trimming [28] and splicing are needed, sometimes refinement may result in superfluous control points. Although T-spline [29] is a good way to obtain a gap-free connection between multiple NURBS surfaces and to perform local refinement efficiently, the construction of the T-spline basis function for the whole solid structure with complex geometry and constituents is still a challenge work.

In this paper, several advanced techniques, including the SBFEM, the NURBS enhanced SBFEM and the polygon elements are jointly used to improve the stress analysis. We define it as the scaled boundary polygon element (SBP) method. Various strategies which help to enhance the accuracy and efficiency of the of stress analysis, such as mesh refinement, nodal enrichment, appropriate placing of the scaling centers, merging excessive polygon elements, NURBS enhanced SBFE model for curved boundary etc. are extensively studied through numerical examples where emphasis is placed on the convergence rate and the computational effort, or the number of nodes used. Based on these studies, further insight in refinement and improvement strategies for stress analysis can be provided.

This paper is organized as follows. In Section 2, an overview of the SBFEM, generation of polygon mesh and the basics of NURBS is presented. In Section 3, NURBS enhanced SBP and subdivision strategies are proposed. Section 4 devoted to study general approaches in improving the accuracy of SBP, including refining polygon mesh, inserting additional nodes on the sides of polygons and appropriately placing the scaling center etc. Numerical examples are provided to validate the rate of convergence for various cases. In Section 5, other strategies are explored to solve typical problems arise in engineering practice. These may be the case containing openings inside the domain with curved boundary, the case containing reentrant corners, the case containing unbounded domain, the case containing edge notch etc. In all the cases, standard SBP are not effective. Measures employing NURBS enhanced SBP and merging polygon elements are recommended. Numerical examples validate the proposed approaches. Finally, in Section 6 the main conclusions are summarized.

2. Summary of SBFEM, polygon mesh and basic properties of NURBS

2.1. Summary of SBFEM

The transformation from Cartesian coordinates (x,y) to the scaled boundary coordinates (ξ,η) is the basic concept of scaled boundary finite element method [8]:

$$\begin{cases} x(\xi,\eta)=\xi x(\eta)+x_0=\xi([N(\eta)]\{x\}-x_0)+x_0 \\ y(\xi,\eta)=\xi y(\eta)+y_0=\xi([N(\eta)]\{y\}-y_0)+y_0 \end{cases} \tag{1}$$

where (x_0,y_0) is the coordinates of the scaling center. As shown in Fig. 1, the dimensionless radial coordinate ξ is pointing from the scaling center O to a point on the boundary with $\xi=0$ at O and $\xi=1$ on the boundary; $[N(\eta)]$ denotes the shape functions for boundary discretization.

Thus the Jacobian matrix and partial derivatives with respect to transformed coordinates (ξ,η) are expressed as

$$\left[\hat{J}\right]=\begin{bmatrix} x_{,\xi} & y_{,\xi} \\ x_{,\eta} & y_{,\eta} \end{bmatrix}=\begin{bmatrix} x(\eta) & y(\eta) \\ \xi x(\eta)_{,\eta} & \xi y(\eta)_{,\eta} \end{bmatrix} \tag{2}$$

$$\left\{ \partial/\partial x \quad \partial/\partial y \right\}^T=\left[\hat{J}\right]^{-1}\left\{ \partial/\partial\xi \quad \partial/\partial\eta \right\} \tag{3}$$

Substituting Eq. (3) into the linear differential operator

$$[L]=\begin{bmatrix} \partial/\partial x & 0 & \partial/\partial y \\ 0 & \partial/\partial y & \partial/\partial x \end{bmatrix}^T=\left[b^1\right]\partial/\partial\xi+\frac{1}{\xi}\left[b^2\right]\partial/\partial\eta \tag{4}$$

with $\left[b^1\right]=\frac{1}{|J|}\begin{bmatrix} y(\eta)_{,\eta} & 0 & -x(\eta)_{,\eta} \\ 0 & -x(\eta)_{,\eta} & y(\eta)_{,\eta} \end{bmatrix}^T$ and $\left[b^2\right]=\frac{1}{|J|}\begin{bmatrix} -y(\eta) & 0 & x(\eta) \\ 0 & x(\eta) & -y(\eta) \end{bmatrix}^T$. $[J]$ is the Jacobian matrix on the boundary and equals

$$[J]=\begin{bmatrix} 1 & 0 \\ 0 & 1/\xi \end{bmatrix}\left[\hat{J}\right] \tag{5}$$

On the other hand, the displacement at a point (ξ,η) is interpolated from the displacement function $\{u(\xi)\}$ on the radial lines

$$\{u_h(\xi,\eta)\}=[N(\eta)]\{u(\xi)\} \tag{6}$$

Substituting Eqs. (4) and (6) into geometric equation $\{\varepsilon\}=[L]^T\{u\}$, the strain vector takes the form of

$$\{\varepsilon\}=\left(\left[b^1\right]\partial/\partial\xi+\frac{1}{\xi}\left[b^2\right]\partial/\partial\eta\right)[N]\{u(\xi)\}=\left[B^1\right]\{u(\xi)\}_{,\xi}+\frac{1}{\xi}\left[B^2\right]\{u(\xi)\} \tag{7}$$

with $\left[B^1\right]=\left[b^1\right][N]$ and $\left[B^2\right]=\left[b^2\right][N]_{,\eta}$.

Substituting Eq. (7) and physical equations $\{\sigma\}=[D]\{\varepsilon\}$ into equilibrium equations $[L]^T\{\sigma\}=0$. And employing virtual work principle

$$\int_V \delta\{\varepsilon\}^T\{\sigma\}dV=\int_0^1 \delta\{u(\xi)\}_{,\xi}^T\left(\left[E^0\right]\xi\{u(\xi)\}_{,\xi}+\left[E^1\right]^T\{u(\xi)\}\right)d\xi$$
$$+\int_0^1 \delta\{u(\xi)\}^T\left(\left[E^1\right]\{u(\xi)\}_{,\xi}+\left[E^2\right]\{u(\xi)\}/\xi\right)d\xi=0 \tag{8}$$

where $\left[E^0\right]$, $\left[E^1\right]$ and $\left[E^2\right]$ are coefficient matrixes related to the geometry and material properties of the medium

$$\left[E^0\right]=\int_{-1}^{+1}\left[B^1(\eta)\right]^T[D]\left[B^1(\eta)\right][J(\eta)]d\eta$$
$$\left[E^1\right]=\int_{-1}^{+1}\left[B^2(\eta)\right]^T[D]\left[B^1(\eta)\right][J(\eta)]d\eta$$