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Dynamic 2.5-D green's function for a poroelastic half-space

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ABSTRACT

The dynamic two-and-a-half-dimensional (2.5-D) Green's function for a poroelastic half-space subject to a point load and dilatation source is derived based on Biot's theory, with the consideration of both a permeable surface and an impermeable surface. The governing differential equations for the 2.5-D Green's function are established by applying the Fourier transform to the governing equations of the three-dimensional (3-D) Green's function. The dynamic 2.5-D Green's function is derived in a full-space using the potential decomposition and discrete wavenumber methods. The surface terms are introduced to fulfil the free-surface boundary conditions and thereby obtain the dynamic 2.5-D Green's function for a poroelastic half-space with the permeable and impermeable surfaces. The half-space 2.5-D Green's function is verified through comparison with the 2.5-D Green's function regarding an elastodynamic half-space and the 3-D Green's function for a poroelastic half-space. A numerical case is provided to compare between the full-space solutions and the half-space solutions with two different sets of free-surface boundary conditions. In addition, a case study of efficient calculation of vibration from a tunnel embedded in a poroelastic half-space is presented to show the application of the 2.5-D Green's function in engineering problems.

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1. Introduction

Many materials encountered in engineering, such as water-saturated soils, oil-impregnated rocks, and air-filled foams, can be considered as saturated porous media. The dynamic responses of a saturated porous medium to internal sources are of great significance to several disciplines, including geotechnical engineering, seismology, and geophysics. The boundary element method (BEM) based on Green's function represents a powerful tool for the study of these problems, especially when the calculation domain of the problem is infinite or semi-infinite. Moreover, when three-dimensional (3-D) loads such as arbitrarily directed incident plane waves, point loads, distributed loads or moving loads are applied to an infinitely long structure with a uniform cross-section, the two-and-a-half-dimensional (2.5-D) BEM and the coupling of the 2.5-D BEM with other methods provide more efficient tools [1–7]. Although the structure can be considered as two-dimensional (2-D), the responses of the structure due to various types of loading will be 3-D; thus, the problem can be regarded as being 2.5-D.

When solving the above-mentioned problem with BEM, the first step is to derive the Green's function. The dynamic Green's function for saturated porous media has been investigated in the previous research. Burridge and Vargas [8] first published the 3-D Green's function for a poroelastic full-space subject to an impulsive point force. Norris [9] obtained the 3-D fundamental solutions for a time-harmonic point force in the solid skeleton as well as a time-harmonic point force in the pore fluid and proposed a closed-form solution in the time domain associated with an impulsive point load applied in a non-dissipative medium. Later, Bonnet [10] and Cheng et al. [11] derived the full-plane (2-D) and full-space (3-D) Green's functions for a time-harmonic point force and dilatation source via analogy with thermoelasticity, respectively. Zimmerman and Stern [12] also obtained the full-space fundamental solutions in the frequency domain for a point force and dilatation source using the potential decomposition method. Lu et al. [5] presented a 2.5-D fundamental solution for a poroelastic full-space using the Fourier transform and the potential decomposition method.

Most geophysical applications involve an infinite boundary with zero traction, which is normally used to model the traction-free surface. Therefore, the half-space fundamental solutions for saturated porous media are more desirable for solving practical problems. Senjuntichai and Rajapakse [13] investigated the 2-D fundamental solutions for a periodic line source buried in a poroelastic half-plane. Philippacopoulos [14] and Jin et al. [15] derived the 3-D fundamental solutions for a vertical and horizontal point

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force, respectively, buried in a poroelastic half-space in the frequency domain. Moreover, Zheng et al. [16] obtained the 3-D Green's function of a poroelastic half-space with the permeable surface for an internal point load and fluid source in a cylindrical coordinate system. In addition, Zheng et al. [17] used the propagator matrix method to obtain the dynamic 3-D Green's functions in a multi-layered poroelastic half-space with the permeable surface.

Summarising, the dynamic 3-D Green's functions subject to a point load and dilatation source buried in a poroelastic full-space, half-space and even layered half-space have been already available for engineering practice. As aforementioned, the 2.5-D approaches are more efficient than 3-D approaches, however, the existing dynamic 2.5-D Green's function has only been derived in a full-space [5]. When the BEM based on the full-space Green's function is used to analyse 3-D dynamic poroelastic problems in a half-space, the free surface of the half-space must be discretised with boundary elements. As a result, two problems arise: First, the increased number of elements will consume large amounts of memory and CPU time. Second, the meshing of the infinite free surface of the half-space requires mesh truncation, which can lead to significant errors near the edges of the model [3]. Nevertheless, these two problems can be dealt with the half-space Green's function implemented in the BEM. Therefore, the dynamic 2.5-D Green's function for a poroelastic half-space may represent a more efficient and precise approach for solving 3-D dynamic poroelastic problems in a half-space by means of the 2.5-D BEM.

In the present paper, the dynamic two-and-a-half-dimensional Green's function for a poroelastic half-space with a permeable surface and an impermeable surface is developed. The dynamic 2.5-D Green's function corresponds to the solutions for the following two problems: an internal point load applied to the solid skeleton and a dilatation source applied within the pore fluid. First, the governing differential equations (**u**–p formulation) of the 2.5-D Green's function for saturated porous media are derived based on Biot's theory and the Fourier transform. Then, a method of displacement potentials is developed to decompose the homogeneous wave equations of the **u**–p formulation into two scalar and one vector Helmholtz equations, through which the general solutions are yielded. Using the discrete wavenumber method, the dynamic 2.5-D Green's function is derived in a poroelastic full-space. Note that the mathematical derivations of the proposed full-space 2.5-D Green's function here has the following ameliorated features, compared to Lu's method [5]. First, two scalar and one vector potentials are used to decompose the homogeneous wave equations subject to an internal point load, in contrast to Lu's study. Second, the discretization of the wavenumber over the horizontal coordinate x is performed in this study. Therefore, it is simple for the present method to reach the half-space 2.5-D Green's function through finding the surface terms that satisfy the free-surface boundary conditions.

As aforementioned, the surface terms are introduced to fulfil the boundary conditions at the surface of the half-space, and both a permeable surface and an impermeable surface are considered. The superposition of the contributions of the full-space 2.5-D Green's function and the surface terms yields the half-space 2.5-D Green's function for saturated porous media. The result of an extreme case for the new 2.5-D Green's function with a vanishing porosity is compared with that of the existing half-space 2.5-D Green's function for an elastodynamic problem. After re-transforming the 2.5-D Green's function into the space domain, the derived half-space 2.5-D Green's function is compared with the existing 3-D Green's function for poroelastic half-space. In addition, a numerical case is provided to compare between the full-space solutions and the half-space solutions with two different kinds of free-surface boundary conditions. In order to show the application of the 2.5-D Green's function in engineering problems, a case study of calculating vibration from a tunnel embedded in a poroelastic half-space is also presented using the proposed half-space Green's function along with 2.5-D boundary integral equation for saturated porous media [6] and the modified Pipe-in-Pipe (PiP) model in which the soil is modelled as the saturated porous medium [18].

2. Governing equations and general solutions

2.1. Biot's theory: governing equations

Based on Biot's theory [19–22], the constitutive equations of dynamic poroelasticity are expressed as

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\varepsilon} + \lambda e\mathbf{I} - \alpha p\mathbf{I} \tag{1a}$$

$$p = -\alpha M e + M\zeta \tag{1b}$$

where

$$e = \nabla \cdot \mathbf{u}, \qquad \zeta = -\nabla \cdot \mathbf{w} \tag{1c}$$

Using a superimposed dot to denote the time derivative and an asterisk "*" to denote the time convolution, the motion equations are

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{F} = \rho_b \ddot{\mathbf{u}} + \rho_f \ddot{\mathbf{w}} \tag{2a}$$

$$-\nabla p + \mathbf{f} = \rho_f \ddot{\mathbf{u}} + m\ddot{\mathbf{w}} + \frac{\eta}{k}K(t) * \dot{\mathbf{w}} \tag{2b}$$

The strain–displacement relationship is

$$\boldsymbol{\varepsilon} = 1/2(\mathbf{u}\nabla + \nabla\mathbf{u}) \tag{3}$$

where

$\boldsymbol{\sigma}$: total stress tensor;
$\boldsymbol{\varepsilon}$: average strain tensor;
p: pore pressure;
e: dilatation of the solid skeleton;
ζ: volume of fluid injected into a unit volume of the bulk material;
u: average skeleton displacement;
w: average fluid displacement relative to the solid skeleton;
I: second-order identity tensor;
F: body forces experienced by the saturated porous medium;
f: body forces experienced by the pore fluid;
λ, μ: solid skeleton Lamé constants, μ is the shear modulus of the bulk material, $\lambda = 1/K - 2/3\mu$ (K: the coefficient of the jacketed compressibility);
α, M: Biot's parameters, $\alpha = 1 - \delta/K$ (δ: the coefficient of the unjacketed compressibility), $M = 1/(\gamma + \delta - \delta^2/K)$ (γ: the coefficient of fluid content for the unjacketed test);
η: viscosity of the pore fluid;
k: permeability of the saturated porous medium;
$\rho_b = (1-\phi)\rho_s + \phi\rho_f$: density of the bulk materials (ρ_s: density of solid skeleton, ρ_f: density of pore fluid, ϕ: porosity);
$m = a_\infty \rho_f / \phi$ (a_∞: tortuosity); and
$K(t)$: time-dependent viscosity correction factor, which describes the transition between the viscous flow in the low-frequency range and the inertia-dominated flow in the high-frequency range.

λ, μ, α and M are the elastic coefficients of the material, Biot and Willis [23] have proposed experiments to measure four quantities from which, in turn, the coefficients λ, μ, α and M may be determined. These measured quantities are μ, K, δ *and* γ . Hughes and Cooke [24] described measurements of porosity ϕ in jacketed compression tests. In addition, Detournay and Cheng [25] listed the poroelastic coefficients for various materials.

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