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Singularity analysis of planar cracks in three-dimensional piezoelectric semiconductors via extended displacement discontinuity boundary integral equation method

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ABSTRACT

The displacement discontinuity boundary integral equation method is extended to analyze the singularity of near-border fields of the planar crack of arbitrary shape in the isotropic plane of a three-dimensional transversely isotropic piezoelectric semiconductor. The hyper-singular boundary integral equations are derived in terms of the displacement, electric potential and carrier density discontinuities across the crack faces, in which body integrals for the carrier density are introduced. Based on the finite-part integrals, singularity exponents and asymptotic expressions of the crack border fields are obtained. The stress, electric displacement and electric current intensity factors are given in terms of the displacement, electric potential and carrier density discontinuities. Finite element results for penny-shaped and line cracks based on the piezoelectric-conductor iterative method are used to verify the derivations of the intensity factors.

1. Introduction

Piezoelectric materials are usually considered to be non-conducting. In 1960, Hutson [1] discovered the piezoelectric effect in semiconductors. This type of semiconductor is called the piezoelectric semiconductor (PSC). In a PSC, mechanical deformation can induce an electric field and the induced electric field can produce an electric current. White [2] showed that an acoustic wave traveling in a PSC medium can be amplified by the application of a dc electric field. The interaction between mechanical fields and mobile charges in PSCs is called the acoustoelectric effect [2,3]. PSCs have been widely used in various smart structures, and electromechanical devices and systems [4–8]. Ten years ago, Hickernell [9] reviewed the development of PSC devices. Since then, the discovery of properties stemming from piezoelectric-semiconducting coupling in nanowires [10] has inspired the development of many devices; it has created a new field called piezotronics that has attached interests for research and various applications. Very recently, Liu et al. [11] reviewed the fundamental theories of piezotronics and piezo-phototronics and Wen et al. [12] summarized the development and progress in piezotronics.

The properties of PSCs are very sensitive to internal defects such as cracks and cavities in materials and structures [13]. Studying cracks in PSCs not only provides benefits in the design and performance of smart devices, but is also important from the perspective of the theory of fracture mechanics for multi-fields. Yang [14] studied an anti-plane, semi-infinite crack in a PSC of 6mm symmetry and obtained an exact solution for the electromechanical fields around the crack. Employing the Fourier transformation method to reduce the mixed boundary value problem to a pair of dual-integral equations, Hu et al. [15] analyzed a finite mode III crack in a PSC of 6-mm crystals and presented numerical results to show how the fracture behavior affects the semiconducting properties. Sladek et al. [16] conducted a transient dynamic analysis of an anti-plane crack problem in functionally graded PSCs using meshless local Petrov-Galerkin method. They obtained a system of ordinary differential equations for the nodal unknowns involved and noted that the stresses and electric displacement field in functionally graded PSCs exhibit the same

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Nomenclature

σ_{ij}, D_i, J_i components of stress, electric displacement and electric current, respectively, which are called the extended stress

u, v, w, φ, n mechanical displacement, electric potential and carrier density respectively, which are called the extended displacement

$c_{ij}, e_{ij}, \varepsilon_{ij}, \mu_{ij}, d_{ij}$ elastic, piezoelectric, dielectric, electron mobility and carrier diffusion constants

ω_{im}, ϑ_{im}, α_{im}, A_i, B_{mi} material related constants in the Green functions

$\Gamma_j^i, K^i, \Gamma^i, K^i, \Theta^*, \mathrm{T}^*$ corresponding integrands in the outer boundary integrals

p_i, d_n, j_n mechanical tractions, surface charge, surface electric current value, respectively, which are called the extended tractions

Ψ^*, Φ^* the Green function of Laplace equation

$K_{\mathrm{I}}^{\mathrm{F}}, K^{\mathrm{D}}, K^{\mathrm{J}}, K_{\mathrm{II}}^{\mathrm{F}}, K_{\mathrm{III}}^{\mathrm{F}}$ extended stress intensity factors

$\alpha_x, \alpha_y, \alpha_z, \alpha_\varphi, \alpha_n$ singularity exponents

$F_{\mathrm{I}}, F_{\mathrm{D}}, F_{\mathrm{J}}$ normalized extended stress intensity factor

L_{ij} material related constants in the extended displacement discontinuity boundary integral equations

q electronic charge

ψ equivalent electric carrier density

χ equivalent body electric charge

V a finite domain

S^0 outer boundary of V

S an arbitrarily shaped planar crack

$\varnothing$ empty set

S^+, S^- upper and lower crack surfaces

singularities at crack tips as in a homogeneous piezoelectric solid. Using the same method, Sladek et al. [17] solved the in-plane crack problem in PSCs under a transient thermal load and obtained the effect of initial electron density on the intensity factors and energy release rate. Sladek et al. [18] investigated the influence of electric conductivity on intensity factors for cracks in conducting piezoelectric materials and functionally graded conducting piezoelectric materials. The interaction integral method was developed for evaluating the intensity factors in functionally graded conducting piezoelectric materials. Very recently, Fan et al. [19] proposed a piezoelectric-conductor iterative method (PCIM) for fracture analysis of PSCs under combined mechanical loading, electric strength field (or electric displacement) and electric current (or electric carrier density). However, a PSC device in practical applications is generally three-dimensional (3D). To date, a study of 3D fracturing in PSCs has yet to be reported.

As is well known, the boundary element method (BEM) has some advantages over the finite element method (FEM), for example, its discretization of a one-dimensional reduction, as opposed to the domain discretization encountered in FEM. Challenges remain when dealing with fracture problems associated with the two degenerated surfaces of a crack. There are some special techniques to overcome this difficulty. For example, Snyder and Curse [20] applied the special Green functions for a line crack to avoid the discretization of its two sides. Another method is the sub-region method presented by Blandford et al. [21], in which the cracked region is divided into several sub-regions separated along the crack surfaces and the conventional BEM is applied to each sub-region. Hong and Chen [22] presented the dual boundary integral equations method to solve crack problems. This dual system incorporates the displacement and traction boundary integral equations by introducing the hyper-singular equation. Crouch [23] proposed the displacement discontinuity method (DDM) so that discretization is required only on one side of the crack surface; this method has been successfully extended to analyze 3D crack problems in elastic media [24–26], and piezoelectric [27] and magneto-electro-elastic media [28]. In addition, the equivalence of the DDM and BEM for solving crack problems was studied by Hong and Chen [29] and revisited by Liu and Li [30]. With the current state of affairs and motivated by related issues, we extend DDM to cracks in 3D transversely isotropic PSCs. As a first step, we develop the extended displacement boundary integral equation method to analytically study singularity behaviors near the crack border, which is an essential problem in fracture mechanics.

The paper is organized as follows. Basic equations for the PSC are given in Section 2. In Section 3, the extended displacement discontinuity boundary integral equations are established. Section 4 analyzes the singularities around the crack border and gives the intensity factors in terms of the extended displacement discontinuity. Finite element numerical examples are given in Section 5. Finally, conclusions are drawn in Section 6.

2. Basic equations

For the static problem of a homogeneous n-type PSC (with a larger electron concentration than hole concentration) in the Cartesian coordinate system *oxyz* in the absence of body force, the governing equations based on the mechanical equations of equilibrium, Gauss' law of electrostatic, and the conservation of charge can be given as [17]

$$\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{xy}}{\partial y}+\frac{\partial \sigma_{xz}}{\partial z}=0, \frac{\partial \sigma_{yx}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \sigma_{yz}}{\partial z}=0, \frac{\partial \sigma_{zx}}{\partial x}+\frac{\partial \sigma_{zy}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}=0, \tag{1a}$$

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}=-qn, \tag{1b}$$

$$\frac{\partial J_x}{\partial x}+\frac{\partial J_y}{\partial y}+\frac{\partial J_z}{\partial z}=0, \tag{1c}$$

where σ_{ij}, D_i, and J_i with $i, j=x, y, z$, are the components of stress, electric displacement and electric current, respectively, which together are called the extended stress; q and n are the electronic charge and the change in the electron density, respectively.

If the PSC is transversely isotropic with the plane of isotropy in the *oxy* plane and polarization direction along the *z*-axis, the constitutive equations can be expressed as [14,17]

$$\begin{aligned}
\sigma_{xx} &= c_{11}\frac{\partial u}{\partial x}+c_{12}\frac{\partial v}{\partial y}+c_{13}\frac{\partial w}{\partial z}+e_{31}\frac{\partial \varphi}{\partial z},\\
\sigma_{yy} &= c_{12}\frac{\partial u}{\partial x}+c_{11}\frac{\partial v}{\partial y}+c_{13}\frac{\partial w}{\partial z}+e_{31}\frac{\partial \varphi}{\partial z},\\
\sigma_{zz} &= c_{13}\frac{\partial u}{\partial x}+c_{13}\frac{\partial v}{\partial y}+c_{33}\frac{\partial w}{\partial z}+e_{33}\frac{\partial \varphi}{\partial z},\\
\sigma_{yz} &= c_{44}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)+e_{15}\frac{\partial \varphi}{\partial y},\\
\sigma_{zx} &= c_{44}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+e_{15}\frac{\partial \varphi}{\partial x},
\end{aligned}$$