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Localized radial basis function scheme for multidimensional transient generalized newtonian fluid dynamics and heat transfer

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ABSTRACT

The local radial basis function (RBF) scheme is developed to simulate 2D and 3D heat transfer and flow dynamics of generalized Newtonian fluids (GNF). The local RBF scheme is a meshless numerical method based on radial basis functions with localized technique. The procedure of localization reduces the computational cost more efficiently than the traditional global RBF method. This meshless method does not require mesh generation, numerical integration and only needs point collocation. Besides, it is very easy to interpolate physical values and its derivatives everywhere in the domain. We consider one isothermal and three non-isothermal multidimensional transient GNF fluid and heat problems in this paper. The dynamic viscosity of the GNF is specified as two different models: the power law model (temperature independent) or Cross model (temperature dependent). The viscous heating is also considered in this work. Numerical results show that the local RBF scheme is stable and accurate as far as the four tested cases are concerned.

1. Introduction

Non-Newtonian or generalized Newtonian fluids are ubiquitous, such as butter, some biological fluids, especially the high molecular weight polymers which are used for our commodity, high-quality electronics, consumer products, and so on. The phenomena of non-Newtonian fluids are very complex and in general non-linear in nature as well. In engineering technology, we generally use numerical methods with generalized Newtonian fluids model to simplify the problem and approximate the flow motion.

There are many existing literature for numerical solutions presented to approximate the problems of flow dynamics and heat transfer in generalized Newtonian fluids. In steady simulations, Bell and Surana [1] presented a p-version least squares finite element formulation for steady 2D isothermal and non-isothermal flows. The p-version least squares finite element formulation was used to approximate the steady-state solution of fully developed flows between parallel plates, flow in symmetric sudden expansions and lid-driven cavity. Bao [2] used an economical finite element scheme to solve the steady-state solutions of backward-facing step and four-to-one abrupt contraction flows. In transient simulations, Neofytou [3] used a third order upwind finite volume method to simulate the unsteady lid-driven cavity flow of GNF. The unsteady incompressible Navier-Stokes equations in primitive variables $(p, \mathbf{v})$ are discretized by the semi-implicit method for pressure linked equation (SIMPLE). Han and Li [4] developed a finite element method with interactive stabilized fractional step Crank-Nicolson based split (CNBS) scheme for non-isothermal flows. They used the CNBS to discretize and solve the momentum-mass conservation equations and also applied the characteristic-Galerkin (CG) method to solve the energy equation. Vaz Jr. and Zdanski [5] presented a fully implicit finite difference method for polymer melt flow and heat transfer. Then Zdanski and his collaborators extended their scheme to 2D and 3D non-isothermal polymer melt flow in sudden expansions [6,7].

Although those researches show good performances in simulating the GNF flows [8–10], the numerical methods adopted are all mesh-dependent methods which need time-consuming mesh generations. The present work proposes a meshless radial basis function (RBF)-based method for 2D and 3D unsteady GNF flows. The merits of the meshless method such as present RBFs collocation method require neither mesh nor relationship of mesh topology. Therefore, it is very suitable to deal with moving boundary problems or irregular geometry especially for the three-dimensional engineering problems. The concept of the RBF-based scheme is that no mesh topology is needed but only the distance of points in the computational domain, so that extension to higher dimensions is only a straightforward procedure not like tedious conventional methods. Further to interpolate the accurate physical values and its higher derivatives in the domain become very easy. Besides it belongs to the high-order approximation scheme so that few points are needed. This is the advantages of meshless RBFs

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Nomenclature

$\mathbf{v}$	velocity
p	pressure
$\boldsymbol{\tau}$	viscous stress tensor
$\dot{\boldsymbol{\gamma}}$	rate of deformation tensor
η	viscosity
$\dot{\gamma}$	shear rate
n	power law index
T	temperature
ρ	density
C_p	specific heat capacity
k	thermal conductivity
Re	Reynolds number $\mathrm{Re} = \frac{\rho V_{ref} L_{ref}}{\eta_{ref}}$
Pr	Prandtl number $\mathrm{Pr} = \frac{C_p \eta_{ref}}{k}$
Pe′	Péclet number $\mathrm{Pe}' = \frac{\rho C_p L_{ref} V_{ref}}{k}$
Br	Brinkman number $\mathrm{Br} = \frac{\eta_{ref} V_{ref}^2}{k(\Delta T_{ref})}$
V_{ref}	reference velocity
L_{ref}	reference length
η_{ref}	reference viscosity
ΔT_{ref}	reference temperature difference
∇^2	Laplacian
$\mathbf{L}$, $\mathbf{B}$	linear elliptic partial differential operators
Ω	computational domain
Γ	boundary
N	number of collocation nodes for global RBF scheme
N_L	number of local supporting nodes for local RBF scheme
c	shape parameter
$\boldsymbol{\alpha}$, α_j	weighting coefficients

over low-order polynomial approximations such as the mesh-dependent FDM, FEM and FVM. These features make the meshless RBFs method very attractive to the numerical modelers. The advantages of modeling by RBFs collocation method will be supported by the following sections in the text. Until now, only few researches focus on the application of the RBF-based scheme for non-Newtonian flows. Bernal and Kindelan [11] used global RBF scheme for the injection problem of non-Newtonian Hele-Shaw flow. They have two important assumptions: (1) creeping flow, the inertia term is ignored; (2) the thermal conduction in flow direction and the thermal convection are neglected since the thickness is small.

The global RBF scheme which considers all global points as supporting nodes induces a highly ill-conditional full matrix. Inverting the full matrix will cost a lot of CPU time and memory, especially for unsteady problems. Lee et al. [12], Tolstykh and Shirobokov [13] and Shu et al. [14] presented more applications of the local RBF scheme. The localized RBF scheme only considers the supporting node in the local region. The system matrix is sparse, so ill-condition problem is overcome and the CPU and memory usages are reduced. Sanyasirajy and Chandhini [15] applied the local RBF scheme to solve 2D unsteady incompressible Newtonian viscous flow in primitive variables formulation $(p, \mathbf{v})$ and then Sevens et al. [16] extended to 3D convective-diffusion problems. In the same trend, Mai-Duy and Tanner [17] employed the integrated RBF networks to compute the non-Newtonian fluid flow problems. Osswald and Hernández-Ortiz [18] presented the RBF method for steady non-Newtonian heat transfer flow problem with 2.5D solution.

In this paper we will use the meshless local RBF scheme to simulate 2D and 3D unsteady, incompressible GNF flow dynamics and heat transfer problems. The governing equations are based on the conservation laws of mass, momentum and energy. The Navier–Stokes equations (the equations of conservations of mass and momentum) are formulated in primitive variables $(p, \mathbf{v})$. We use the local RBF scheme with the Chorin pressure projection method (PPM) [19] in 1968 or the Patankar and Spalding SIMPLE [20] in 1972 to discretize the pressure-velocity coupling equations. The GNF dynamic viscosity is defined by the power law model (temperature independent) or Cross model (temperature dependent).

The governing equations for the incompressible GNF thermal flow are mass, momentum and energy conservation equations and are described in Section 2. Section 3 gives the details of local RBF scheme for solving computational fluid dynamics and heat transfer problems and how to interpolate numerical solutions of physical values and their derivatives by the local RBF scheme. In Section 4 we carry out the validation of the local RBF scheme by solving four benchmark GNF problems (lid-driven cavity flow, non-isothermal Poiseuille flow, 2D and 3D non-isothermal backward-facing step flow). Numerical results for these problems are compared with the results found in the literature. Section 5 is the conclusions based on the numerical results.

2. Governing equations

In present mathematical model, we consider the unsteady flow phenomena of incompressible GNF. No body forces are taken into account. The governing equations to be solved are mass and momentum conservation equations [21],

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\rho \frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -\nabla p + [\nabla \cdot \boldsymbol{\tau}], \tag{2}$$

where, ∇ is the gradient operator, $\mathbf{v}$ is the velocity vector, $\mathrm{D}/\mathrm{D}t$ is the material derivative, ρ is the density, p is the pressure, and $\boldsymbol{\tau}$ is the viscous stress tensor.

The viscous stress of the incompressible generalized Newtonian fluid is defined by the constitutive equations:

$$\boldsymbol{\tau} = \eta \dot{\boldsymbol{\gamma}}, \tag{3}$$

where η is the dynamic viscosity and $\dot{\boldsymbol{\gamma}} = \nabla \mathbf{v} + \nabla \mathbf{v}^T$ is the rate of deformation tensor. In Newtonian fluid, the viscosity η is a constant. In GNF, the viscosity η can be expressed as a function of shear rate or a function of shear rate and temperature. In this paper, we describe the viscosity η by the power law model as well as the Cross model:

For the power law model:

$$\eta = \eta(\dot{\gamma}) = m_0 \dot{\gamma}^{n-1}, \tag{4}$$

where m_0 is the consistency coefficient (with the dimension $\mathrm{Pa} \cdot \mathrm{s}^n$), n is the power law index (dimensionless), Eq. (4) reduces to Newtonian law of viscosity for $n = 1$. The shear rate $\dot{\gamma}$ (also called second invariant) is defined as:

$$\dot{\gamma} = \sqrt{\frac{1}{2} \dot{\boldsymbol{\gamma}} : \dot{\boldsymbol{\gamma}}}. \tag{5}$$

For the Cross model:

$$\eta = \eta(T, \dot{\gamma}) = \frac{\eta_0(T)}{1 + (\lambda(T)\dot{\gamma})^{1-n(T)}}, \tag{6}$$

where $\eta_0(T) = a_1 \exp(a_2/T)$, $\lambda(T) = b_1 \exp(b_2/T)$ and $n(T) = c_1 \exp(-c_2/T)$.