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An improved Moving Kriging-based meshfree method for static, dynamic and buckling analyses of functionally graded isotropic and sandwich plates

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A B S T R A C T

A meshfree method with a modified distribution function of Moving Kriging (MK) interpolation is investigated. This method is then combined with a high order shear deformation theory (HSDT) for static, dynamic and buckling analyses of functionally graded material (FGM) isotropic and sandwich plates. A meshfree method uses the normalized form of MK interpolation under a new quartic polynomial correlation to build the basis shape functions in high order approximations. The Galerkin weak form is used to separate the system equations which is numerically solved by meshfree method. A rotation-free technique extracted from isogeometric analysis is introduced to eliminate the degrees of freedom of slopes. Then, the method retains a highly computational effect with a lower number of degrees of freedom. In addition, the requirement of shear correction factors is ignored and the traction free is at the top and bottom surfaces of FGM plates. Various thickness ratios, boundary conditions and material properties are studied to validate the numerical analyses of the rectangular and circular plates. The numerical results show that the present theory is more stable and well accurate prediction as compared to three-dimensional (3D) elasticity solution and other meshfree methods in the literature.

1. Introduction

Functionally graded materials (FGMs) were first introduced by a group of Japanese scientists in the last two decades [1,2]. Until now, FGMs have been developed and widely utilized in various engineering fields. FGMs have played an important role and made preferable many disciplines such as shipbuilding, automotive engineering, nuclear power planning and aero-space, medical devices, etc. In this research, the concerned FG material is used to study since the gradual change of its properties can be tailored to many applications and working environments. In the typical FGM plates, the material is continuously changed in microstructure from ceramic in top layer and metal in bottom one. It is known that the ceramic with a low conductivity can well resist to the effects of thermal stress due to the high temperature environment as well as the surface corrosion of components [3–5], meanwhile the metal is well capable of imposing high loading intensity to the structures. Research studies for the thermal stress analysis of FGMs were reported in [6,7]. Other applications of FGMs can be found in electronic fields [8,9]. In addition, the transient waves by exciting impact loads in plates are very promising in terms of non-destructive evaluation and material characterization [10–12], etc. To use these materials effectively, static, dynamic and buckling analyses are really necessary and therefore the research task has never been ended.

Besides various applications of FGM plates to engineering problems, many plate theories have been proposed such as the classical plate theory (CPT) [13,14] with Kirchhoff–Love assumptions for thin plate. The first order shear deformation (FSDT) [15–17] which included shear effects was then proposed for Reissner–Mindlin plates. However, with the assumption of linear in-plane displacement, the results reported in [18] showed that the zero-shear strain/stress condition has been not satisfied on the plate surfaces and it can lead to inaccurate results. It was also noted that the presence of shear correction factors (SCFs) [19] aims to adjust the unrealistic shear strain component of energy equation which occurs in plate and shell issues. However, such adjustment can lead to complicated computation and inaccuracy results due to the linear distribution of

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shear components through the plate thickness. To overcome these drawbacks, numerous researches of higher-order shear deformable theory (HSDT), which focus on the justification of distribution function $f(z)$, have been addressed [20,21]. It is found that better numerical results can be achieved as pointed out in many reports such as third-order shear deformation theory (TSDT) [21,22], fifth-order shear deformation theory (FiSDT) [23,24], trigonometric shear deformation theory [25–28], exponential shear deformation theory (ESDT) [29–31] and so on. More importantly, the requirement of HSDT theory is C^1-continuous, which is not able to implement in the standard finite element method. There are some improvements for consecutive condition by using the C° continuous elements or classical Hermite interpolations which involve in transverse and rotations [32,33]. However, the increasing unknown variables are inevitable [32] and arise high computational costs. In this paper, a modified Moving Kriging interpolation function [34,35] is implemented into the meshfree method and the present method does not require any additional variables.

In the framework of numerical methods, many meshless methods [36–54] have been shown as efficient computational tools for engineering problems. They were named as element-free Galerkin method (EFG) [36], the reproducing kernel particle method (RKPM) [37], the smoothed particle hydrodynamics (SPH) [38,39], the meshless local Petrov–Galerkin (MLPG) [40,41], etc. Among them, the element-free Galerkin method which employed the moving least squares (MLS) approximation [42,43] was very widely used. However, the major difficulty is that the moving least square function (MLS) does not satisfy the Kronecker delta condition [44]. Hence, the special treatment of the essential boundary condition needs to be provided such as penalty method [48], Lagrange multipliers [49], Transformation method [50]. More importantly, an improved meshfree method with radial point interpolation function [51–54] used the smoothed technique as found in Refs. [55–60] can resolve the adverse condition of Kronecker delta. This will be a great motivation for our forthcoming study. On the other hands, an alternative approach of a combination of Galerkin weak form and Moving Kriging (MK) interpolation function [61] allows us to overcome Kronecker's delta property. Such a method with the essential boundary conditions is easily enforced in a similar way as the conventional finite element method (FEM). However, the MK distribution interpolation based on the original Gaussian exponential function depends on the correlation parameter θ [62]. The evaluation of MK shape function is accurately determined by choosing an arbitrarily correlation factor on a bound of $0.1 \div 500$ and yet any publications to search an optimal value θ. This causes the instability in solution and even undetermined solution in several cases.

In this work, we investigate a modified distribution of MK shape function and numerically show that the proposed method is insensitive to the correlation parameter θ. By a minor justification, the present formulation is significantly improved with high accuracy. In addition, the in-plane displacement is written in a compact form of higher order displacement field and constant transverse deflection through the plate thickness. Numerical study points out that the present method with the normalization of MK distribution function via quartic polynomial function can produce more stable solutions than the published approach [62]. The power and Mori–Tanaka models are utilized to homogenize the material properties. Numerical examples of isotropic and sandwich FGM plates are shown to validate high performance of the proposed method.

2. The higher order shear deformation theory for FGM plates

2.1. Problem formulation

A rectangular plate is illustrated with dimension of a, b and plate thickness h as shown in Fig. 1. Three functionally graded material (FGM) plates typically consist of isotropic plates (type 1), sandwich plates using with FGM core and isotropic skins (type 2) and vice versa (type 3).

2.1.1. Isotropic FGM plates (type 1)

The FGM plate is composed of two different material phases which are ceramic and metal arranging at the top and bottom of the plate, respectively. The effective moduli of two constituents are homogenized by the rule of mixture or the Mori–Tanaka models which are used to evaluate the effective elastic properties of the grade composite. The volume fraction of two distinct phases is described as [32]

$$V_c(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^n, \; z \in [-h/2, h/2]\,; \quad V_m = 1 - V_c \tag{1}$$

where m and c are defined to the metal and ceramic. The power index n is representative to the volume fraction varying through the thickness. The rule of mixture [32] used to estimate the effective moduli of material is defined as

$$P_e = P_c V_c(z) + P_m V_m(z) \tag{2}$$

where P_c, P_m represent the material constituents of the ceramic and the metal, respectively. Young's modulus (E), Poisson's ratio (ν), and density (ρ) can follow the rule defined by Eq. (2).

However, the rule of mixture does not reflect the interactions among the two materials [63,64]. Meanwhile, the Mori–Tanaka model [65] is assumed to calculate their interactions through the effective bulk and shear modulus given by

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m \frac{K_c - K_m}{K_m + 4/3\mu_m}}\,; \quad \frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m \frac{\mu_c - \mu_m}{\mu_m + f_1}} \tag{3}$$

where

$$f_1 = \frac{\mu_m (9K_m + 8\mu_m)}{6(K_m + 2\mu_m)} \tag{4}$$

The effective Young's modulus E_e and Poisson's ratio ν_e are respectively now written as

$$E_e = \frac{9K_e\mu_e}{3K_e + \mu_e}, \qquad \nu_e = \frac{3K_e - 2\mu_e}{2(3K_e + \mu_e)} \tag{5}$$

The comparison of the effective Young's modulus corresponding with various power index values of Al/ZrO_2 FGM plate between the mixture and Mori–Tanaka schemes is illustrated in Fig. 2. For the homogeneous constituents, the identical values are given by two models. However, when the material is inhomogeneous, the effective moduli of the Mori–Tanaka scheme is lower than that of the rule of mixture one.

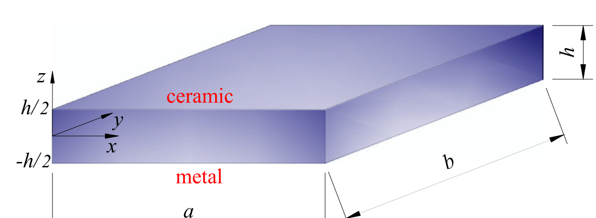

Fig. 1. A typical FGM plate.