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Dual boundary element analysis of fatigue crack growth, interaction and linkup

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ABSTRACT

Multiple-site and widespread fatigue damage have been an issue to the aircraft and construction industry for a long period. Structural components develop cracks at several locations which grow with crack paths that are difficult to predict. When two cracks approach one another, their stress fields influence each other leading to an enhancing or shielding effect which depends on the position and orientation of the cracks. Since there are no generalized analytical methods for predicting crack stress fields, simulation of multiple-crack growth is an important and challenging task which is still an evolving area of research.

This paper describes a two-dimensional application of the dual boundary element method (DBEM) to the analysis of mixed-mode multiple-crack growth in linear elastic fracture mechanics, under fatigue loading. The crack-growth process is simulated with an incremental multiple-crack extension analysis based on the maximum principal stress criterion. For each increment of the analysis, in which crack extensions are modelled with new straight boundary elements, the DBEM is applied to perform a single-region stress analysis of the cracked structure and the J-integral is used to compute the stress intensity factors. The incremental analysis is based on a prediction–correction technique that defines, in each increment of the analysis, the direction and the extension of the multiple interacting cracks, thus taking into account the discreteness of the analysis and ensuring that the requirement of the path uniqueness is satisfied. Based on the ligament yield criterion which assumes that when the plastic zones of two adjacent cracks touch each other, the ligament between the cracks fails and the cracks coalesce, plates with multiple-site damage can be analysed. The fatigue life and residual strength of the structure are introduced as a post-processing procedure on the results of the multiple-crack growth. Results of this incremental analysis are presented for several geometries with multiple-site damage, demonstrating the accuracy and efficiency of the strategies adopted in the analysis.

1. Introduction

Catastrophic fracture failure of engineering structures is caused by cracks that extend beyond a safe size. Cracks, present to some extent in all structures, either as a result of fabrication defects or localized damage in service, may grow. The crack growth leads to a decrease in the structural strength. Thus, when the service loading cannot be sustained by the current residual strength, fracture occurs leading to the failure of the structure. Fracture, the final catastrophic event which takes place very rapidly, is preceded by crack growth which develops slowly during normal service conditions, mainly by fatigue due to cyclic loading.

Linear elastic fracture mechanics can be used to describe the behaviour of the cracks. The fundamental postulate of the linear elastic fracture mechanics is that the crack behaviour is determined solely by the values of the stress intensity factors which are a function of the applied load and the geometry of the cracked structure and thus, play a fundamental role in linear elastic fracture mechanics applications.

Crack-growth processes are simulated with an incremental crack-extension analysis. For each increment of the crack extension, a stress analysis is carried out and the stress intensity factors are evaluated. The paths of the multiple cracks, predicted thus on an incremental basis, are computed through a criterion defined in terms of the stress intensity factors. Crack-tip linkup, of adjacent cracks, is considered in this process.

The geometry of engineering structures, which is continuously changing with the extension of the multiple cracks, requires the use of numerical methods to evaluate the stress intensity factors. The boundary element method is a well-established numerical technique in the engineering community, see Brebbia [1] and Brebbia et al. [2]. The boundary element method has been

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successfully applied to linear elastic problems in domains containing no cracks. For symmetric crack problems only one side of the crack need be modelled and a single-region boundary element analysis may be used. However, in a single-region analysis, the solution of general crack problems cannot be achieved with the direct application of the boundary element method, because the coincidence of the crack boundaries causes an ill-posed problem. Among the techniques devised to overcome this difficulty, the most general are the sub-regions method, presented by Blandford et al. [8] and the dual boundary-element method (DBEM), first presented by Portela et al. [5] in elastostatics. The main drawback of the method of subregions is that the introduction of artificial boundaries, which connect the cracks to the boundary so that the domain is partitioned into subregions without cracks, is not unique and thus it cannot easily be implemented into an automatic procedure to simulate the growth of multiple cracks. On the other hand, the DBEM is the most efficient technique devised to overcome this difficulty. It introduces two independent equations, the displacement and traction boundary integral equations, with the displacement equation applied for collocation on one of the crack surfaces and the traction equation on the other. With this strategy, general mixed-mode crack problems can be solved in a single-region boundary element formulation, with both crack surfaces discretized with boundary elements.

Historically, the use of dual integral equations was first reported by Bueckner [3] in crack problems, by Watson [4] in the boundary element method and by Hong and Chen [6] who derived integral equations of elasticity. However, it is well known in the scientific community that the effective implementation of the dual boundary element method for crack problems was first reported by Portela et al. [5]. A thorough review article of dual boundary element methods, with emphasis on hypersingular integrals and divergent series, was presented by Chen and Hong [7].

Within the limits of linear elastic analysis, the stress field is unbounded at the tip of a crack. This was early reported by Brahtz [11] and later by Williams [12] who, after an investigation of the analytical form of these singularities demonstrated that under all possible combinations of boundary conditions, the stress becomes infinite at the tip of a crack. From a physical point of view, unbounded elastic fields are meaningless. Nevertheless, unbounded stresses cannot be ignored as their presence indicates that new phenomena (e.g. plasticity and fracture) may occur, leading to localized damage in practical situations. In this paper, the term *singularity* is used to denote the cases in which the elastic stress field becomes unbounded. If r denotes the distance measured from the crack tip, the stress field is of the order $r^{-1/2}$ which becomes singular as r tends to zero. The stress intensity factor (SIF), defined at the crack tip, is a measure of the strength of this singularity.

The presence of the stress singularity in the numerical model raises considerable numerical difficulties by virtue of the need of simultaneously representing both the singular and the finite stresses in the numerical model. The performances of the most important approaches that have been devised to overcome this difficulty, in the finite element method (FEM), in the extended finite element method (XFEM), in the boundary element method (BEM) and in meshfree or meshless methods, are briefly reviewed in the following.

A common procedure, used in the very early fracture-mechanics applications of the finite element method, is to ignore the presence of the singularity and to attempt to diminish its effect on the overall solution by using mesh refinement in the neighbourhood of the crack tip. The numerical value of the calculated stress components at the crack tip will always be finite, but it can be made as large as one desires by increasing refinement of the mesh. Obviously, this procedure is mesh dependent and, if it converges, will produce a slow-convergence ratio in the entire domain of the problem, as shown by Tong et al. [18]. This is obviously a consequence of the impossibility of representing simultaneously both the singular and the finite stresses in the numerical model, simply with a mesh refinement procedure. In this approach, the stress intensity factors are evaluated from a correlation procedure, involving a comparison between the numerical results of either the displacement or the stress fields and the respective analytical solutions, represented in the form of an eigenfunction expansion series around the crack tip. Typically, the stress intensity factors obtained by application of this correlation procedure at crack-face nodal points are then extrapolated to the crack tip. Consequently, stress intensity factors cannot be computed accurately only with the mesh refinement procedure. This was shown, for instance, in the work of Portela et al. [5], where values of the stress intensity factors, computed by a displacement correlation procedure, are compared with those values obtained with the J-integral technique, for several crack problems analysed by the dual boundary element method.

The use of quarter-point isoparametric finite elements, introduced by Henshell [40] and Barsoum [41], suggested the application of quarter-point boundary elements at the crack tip, as an alternative to the mesh refinement procedure. However, while quarter-point finite elements both represent the $r^{1/2}$ displacement behaviour and introduce a $r^{-1/2}$ singularity in the stress field, the use of quarter-point elements in the boundary element method, in which displacements and tractions are approximated independently, enables only the displacement behaviour to be properly represented. This feature was early noticed by Cruse et al. [42] who introduced traction-singular quarter-point boundary elements for the correct representation of the singularity in the stress field. Stress intensity factors can be computed from quarter-point elements by the displacement correlation procedure. The application of this procedure over quarter-point elements, first presented by Blandford et al. [43], was called a two-point formula by Smith [44]. The computation of stress intensity factors from traction-singular quarter-point boundary elements was presented by Martinez et al. [45] who have shown that the use of the crack-tip traction nodal values of the singular element is less sensitive to the discretization than any of the displacement correlation procedures. In general, the accuracy of stress intensity factors, computed from quarter-point boundary elements by the displacement correlation procedure, depends on the size of these elements, as reported by Harrop [46] who studied the case of quarter-point finite elements and concluded that it is impossible to recommend a particular size for the quarter-point element, suitable for all situations.

While the above methods represent the stress singularity in the numerical model, an alternative approach, developed by Symm [47] in potential theory, is based on the subtraction of this singularity from the numerical model. In fracture mechanics applications, the singularity subtraction technique is a procedure that uses a singular particular solution of the crack problem to regularize the stress field and to introduce, simultaneously, the stress intensity factors as additional primary unknowns in the problem. This approach was first applied by Xanthis et al. [48] for anti-plane problems and by Aliabadi et al. [14] to solve symmetrical crack problems using the boundary element method. Analysis of symmetrical problems with the singularity subtraction technique is straightforward, because the singular tractions are among the problem unknowns, when only half of the problem is considered with the proper boundary conditions along the symmetry line.

In the case of non-symmetrical problems, the singular tractions are not among the boundary element unknowns and consequently, there is no singularity in the numerical model to be subtracted. The application of the sub-regions boundary element method is an obvious way to circumvent this difficulty, as shown