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Application of meshless procedure for the peristaltic flow analysis

Jakub Krzysztof Grabski *, Jan Adam Kołodziej [1], Magdalena Mierzwiczak [2]

Institute of Applied Mechanics, Poznan University of Technology, Jana Pawła II 24, 60-965 Poznan, Poland

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ABSTRACT

The paper deals with the problem of the peristaltic flow of Newtonian fluid in two-dimensional channel. The problem is considered using stream function and vorticity formulation. The high-order iterative formulation is used in order to transform the nonlinear problem into a hierarchy of inhomogeneous problems which are solved using the method of fundamental solutions and the radial basis functions. The first approximation is obtained for Reynolds number $Re = 0$. In the paper results are presented for different values of Reynolds number and flow rate.

1. Introduction

The term peristalsis comes from Greek word *peristaltikos*, which means clasping and compressing. The peristaltic pumping is understood as a transport of fluid induced by progressive wave of contraction along the distensible duct. The peristaltic flow exists in many parts of our body such as the gastrointestinal tract, the ureter, the lymphatic vessels and the small blood vessels. Peristaltic pumps are also used in industry (to transport corrosive or aggressive fluids) and medicine (used for example for dialysis). A wide range of applications of peristaltic flow phenomena requires a good theoretical basis. Therefore the existing literature on numerical study of peristaltic flow is quite extensive. There is a wide range of models which were used in the literature to study the peristaltic flow.

Many various models of the considered channel or tube were used to the numerical analysis of the peristaltic flow. In many papers the flow was considered in the plane symmetric channel. Some authors concerned the flow in the plane asymmetric channel [1–8]. The peristaltic flow was considered also in the plane incurved channel [9–13]. The peristaltic flow in the axisymmetric channel was considered by some authors [14–18]. Furthermore in some papers the flow was analyzed in the axisymmetric channel with endoscope [19–23]. Akbar and Nadeem investigated the peristaltic flow in the axisymmetric diverging tube [24]. Also the influence of different shapes of the peristaltic wave (sinusoidal, square, trapezoidal, triangular) was investigated [15,22,24]. Walker and Shelley made shape optimization of the peristaltic wave using sequential quadratic programming [25]. The flow in the channel with compliant walls was also investigated in some papers [11,12,26–28].

Various models of the fluid were used in the publications on peristaltic flow. Most authors used viscous, incompressible model of the fluid because of its simplicity. However in recent times more and more attention received the analysis of the non-Newtonian fluid. In papers on the peristaltic flow some authors used Jeffrey fluid model [2,6,22,29,30]. The Williamson fluid model was used in [1,7,31]. In existing literature many another models of fluid were used: Johnson–Segalman fluid [13], Eyring–Powell fluid [24], pseudoplastic fluid [15], Maxwell fluid [32,33], third grade fluid [9,11], fourth grade fluid [34], Carreau fluid [22], Oldroy-B fluid [18], micropolar fluid [35], hyperbolic tangent fluid [21], nanofluid [26], and biofluid with variable viscosity [21]. Some additional assumptions which make the governing equations more complex also appear in the literature on the numerical analysis of the peristaltic flow. The peristaltic flow through porus medium was considered in [8,28,30,36]. Some authors investigated the flow with heat and mass transfer [1,4,8,11,12,20,22,30,37,38]. The influence of external magnetic field on peristaltic flow was analyzed in [1,2,13,14,19,20,22,28,34,35,39,40].

In the publications which deal with the numerical study of peristaltic flow various methods were used in order to solve the governing equation of the problem. Most of authors used the perturbation method, for example in [41,42] or analytic solution under certain assumptions, for example in [43–47]. The finite difference method was used in [48]. Pozrikidis obtained solution using the boundary integral method [49]. Some authors used the

* Corresponding author. Tel.: +48 61 665 2619.
E-mail addresses: jakub.grabski@put.poznan.pl (J.K. Grabski), jan.kolodziej@put.poznan.pl (J.A. Kołodziej), magdalena.mierzwiczak@put.poznan.pl (M. Mierzwiczak).
[1] Tel.: +48 61 665 2321.
[2] Tel.: +48 61 665 2387.

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finite elements method [50,51]. Spectral methods were used in [52,53]. To the best of our knowledge the literature on numerical study of peristaltic flow papers in which the method of fundamental solutions (MFS) was used does not exist.

The MFS belongs to the meshless methods group. The method can be used to solve problems in which the fundamental solutions are known. The approximate solution is a linear combination of fundamental solutions (source function). The fundamental solution is a function of distance the point inside the considered region from the source point. The source points are located on a *pseudo-boundary* outside the region. Boundary of the considered region and the *pseudo-boundary* does not have any common points. The differential equation is satisfied exactly by fundamental solution at any point in the considered region ensuring that also the linear combination of fundamental solutions (the approximate solution) fulfills the governing equation at any point in the region. The boundary conditions are fulfilled approximately by the approximate solution using the boundary collocation method. The MFS was proposed by Georgian scientists [54]. The numerical implementation of MFS was presented by [55].

As yet application of MFS for solution of non-linear boundary value problems (to which Navier–Stokes equations belong) really are not numerous. For the best knowledge of authors the first attempt of MFS's application for non-linear Poisson problem was given in [56]. In this paper the particular solution was expressed by integral through considered region and as the sum of right-hand side function times fundamental solution. Then the Picard iteration method was used. In [57–61] original non-linear Poisson-type differential equation in two-dimensional domain is converted to sequence of linear Poisson equations. Then, the radial basis functions (RBF) and the MFS are used respectively to construct the expression of particular and homogeneous solution on each iteration step. This procedure was applied for some more complicated problems of applied mechanics, namely large deflection of plates [62], isothermal gas flow in porous medium [63], thermoelasticity of functionally graded material [64], determination of effective thermal conductivity of unidirectional composites with linearly temperature dependent conductivity of constituents [65], two-dimensional non-linear elasticity [66], elasto-plastic torsion of prismatic rods [67], and some inverse problems [68,69].

The steady-state heat conduction with temperature-dependent thermal conductivity and mixed boundary conditions involving radiation was investigated using MFS in [70]. Authors employed the classical Kirchhoff transformation. In this way the governing equation was transformed to Laplace equation. After this the only nonlinearity in new boundary value problem was in boundary conditions. After collocation of these boundary conditions the non-linear system of algebraic equations was solved by standard procedure. Another problem with linear governing equation and non-linear boundary conditions for which MFS was applied is water waves problem [71–75].

In papers [76,77] authors proposed the linearization scheme for the nonhomogeneous term in terms of the dependent variable and differencing in time (first or second derivative with respect time) resulting in Helmholtz-type equation whose fundamental solutions are available. Consequently the particular solutions are no longer needed and MFS can be directly applied to linearized equation. The perturbation technique was combined with the MFS to solve nonlinear Poisson-type problem in papers [78,79]. Due to this the nonlinear problem is transformed into a sequence of nonhomogeneous linear ones which can be solved by MFS and RBF. In papers [80,81] the homotopy analysis method was combined with the MFS for solution of non-linear Poisson type problem. For the best knowledge of authors the first trial of MFS's application for Navier–Stokes equations was given in paper [82]. Using operator-splitting scheme the unsteady Navier–Stokes equations were transformed into simple advection–diffusion and Poisson equations. The resultant velocity advection–diffusion equations and the pressure Poisson equation were then calculated using the MFS together with the Eulerian–Lagrangian method and the method of particular solutions.

In the paper we considered the peristaltic flow of Newtonian fluid in plane symmetric channel. We used the high-order iterative algorithm proposed by Zhao and Liao [83]. They solved the Navier–Stokes equations using the algorithm and the boundary element method. In the paper we propose to use the same algorithm in combination with the MFS and the RBF in order to solve the nonlinear boundary value problem. The solution was obtained using the method of particular solution. On each iteration step the general solution was obtained using MFS. The RBF interpolation was used for getting the particular solution.

2. Geometry of the considered problem

Example of the two-dimensional channel for the peristaltic flow is depicted in Fig. 1.

The walls of the channel can be described in two coordinates systems. The first one is stationary fixed system, called the laboratory frame (x', y'). The position of the peristaltic walls described in this coordinates system is time-dependent and represented by:

$$y'(x',t) = \pm\left[h - \varepsilon\cdot\cos\left(\frac{2\pi(x'-ct)}{\lambda}\right)\right], \tag{1}$$

where h denotes average distance between the wall and the symmetry axis of the channel, ε is the amplitude of the peristaltic wall, c is the velocity of the peristaltic wave and λ is the length of the peristaltic wave. The peristaltic flow described in the laboratory frame is unsteady.

The second coordinates system (x, y) (called the wave frame) moves with the same speed as the peristaltic wave c. Thus the walls of the peristaltic wave observed in the wave frame can be defined as:

$$y(x) = \pm\left[h - \varepsilon\cdot\cos\left(\frac{2\pi x}{\lambda}\right)\right]. \tag{2}$$

The main advantage of this system is that the peristaltic flow observed in the wave frame can be treated as steady because positions of the peristaltic walls are fixed. Equations which described the transformations between the laboratory frame and the wave frame are defined as follows:

$$x = x' - ct, \quad y = y'. \tag{3}$$

After introducing dimensionless variables:

$$X = \frac{x}{\lambda}, \quad Y = \frac{y}{\lambda}, \quad E = \frac{\varepsilon}{\lambda}, \quad H = \frac{h}{\lambda}, \tag{4}$$

Eq. (2) takes the following form:

$$Y(X) = \pm[H - E\cdot\cos(2\pi X)]. \tag{5}$$

Finally the considered region Ω with characteristic dimensionless variables is depicted in Fig. 2.

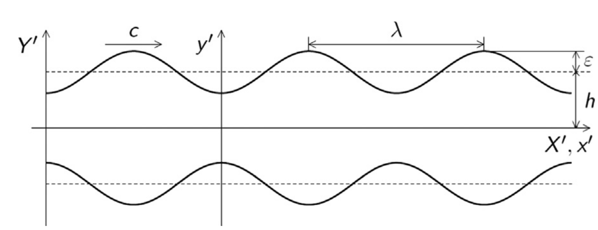

Fig. 1. Geometry of the peristaltic channel.