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Electromagnetic scattering analysis using nonconformal meshes and monopolar curl-conforming basis functions

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ABSTRACT

A scheme for electromagnetic scattering analysis of perfect electric conducting (PEC) objects using nonconformal meshes is developed in this paper. The difference of the integral operators for the electric field integral equation (EFIE) and the magnetic field integral equation (MFIE) are analyzed in detail. It is shown theoretically that basis functions used to expand the surface currents for the MFIE may not necessarily be divergence-conforming. The nonconformal meshes and monopolar $\mathbf{n} \times$ RWG basis functions are used together to solve the MFIE. Details for the implementation of the proposed method are presented. The method is verified through the numerical results for electromagnetic scattering analysis from several PEC objects. It is shown that this method is a suitable choice for using nonconformal meshes when solving electromagnetic scattering problems with the MFIE.

1. Introduction

To solve electromagnetic scattering problem of three-dimensional perfect electric conducting (PEC) objects using method of moments (MoM), the surface of the target in analysis should be firstly discretized with suitable elements. This is a vital step for successful analysis of electromagnetic scattering problems. To model arbitrarily shaped scatterer, triangular elements and the Rao-Wilton-Glisson (RWG) [1] basis function defined on a pair of triangular elements have been used widely in the literature [2–7] since it was proposed in [1]. The RWG function keeps the continuity of the normal component of the expanded electric currents. Thus, to use this kind of basis function correctly, neighboring triangular elements should share one common edge and two common nodes. This means that conformal triangular meshes must be used to support the definition of the RWG functions. For objects with regular shapes, commercial software can be used to generate meshes automatically. However, for some complicated structures with tiny parts or with geometric discontinuities, the generation of meshes with high quality is still a tedious and time-consuming process. For multiscale problems, the electric size of different parts of a complicated object may differ greatly. When conforming meshes are used to discretize the whole object, some parts may be discretized with too dense mesh which makes the total number of unknowns too large.

To deal with the former mentioned problems caused by the use of conformal meshes, several different techniques are proposed in literature. In [8], a novel meshless scheme by applying the Green's theorem to surface integral equations with flat integral domain is developed. In [9], a framework that permits seamless inclusion of multiple functions within the approximation space and applicable to nonconformal tessellations is proposed. However, both methods in [8,9] need special technique to deal with the hyper-singular integrals. Two other methods are proposed in [10,11] to deal with the superfluous radiation of the accumulated charges on boundaries of elements when nonconformal meshes are used to the combined field integral equation (CFIE) and electric field integral equation (EFIE) respectively. The method used in [10] is a discontinuous Galerkin surface integral equation method and it allows the use of square-integrable basis and test functions without any considerations of continuity requirement across element boundaries. In [11], the authors use a ‘even-surface odd-volumetric’ monopolar RWG set discretization of the EFIE. Besides, a volumetric testing over a set of tetrahedral elements are implemented to easy the calculation of the hyper singular integral.

In [10,11], the authors select the monopolar RWG functions as basis functions. However, when nonconformal meshes are used, any suitable basis functions defined on one element can be used as basis functions and the monopolar RWG set is not the only choice. In fact, in a former paper [12], it has been shown that monopolar $\mathbf{n} \times$ RWG set can also be used to the MFIE. However, only conformal meshes have been used in that paper. It should be noticed

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that the monopolar RWG or monopolar $\mathbf{n} \times$ RWG basis functions, or even the standard Galerkin RWG discretization, do not lead to a conforming discretization strategy for the MFIE [13,14]. Consequently, these techniques do not necessarily guarantee convergence of the solution in the norm of the numerical solution space. In this paper, we study the use of the monopolar $\mathbf{n} \times$ RWG set to the magnetic field integral equation (MFIE) using nonconformal meshes. Besides, although basis functions which do not keep the continuity of the normal component of the equivalent surface electric currents have been used to the MFIE in [12,15,16], it has not been shown why these kinds of basis functions can be used to the MFIE since the normal component of the surface electric currents should be continuous in the EFIE. An explanation will be given to this phenomenon in this paper. Based on this explanation, a nonconformal scheme of the MFIE using nonconformal meshes and monopolar $\mathbf{n} \times$ RWG set are developed. Numerical results are also presented to validate the proposed nonconformal scheme.

2. Mathematical constraints of integral operator on the choice of basis functions

For the analysis of electromagnetic scattering problems using integral equation method, the mathematical property of the integral operator imposed on the equivalent electric currents plays an important role in the choice of the basis functions. The basis functions used in MoM should be consistent with the corresponding mathematical constraints inherited from the integral equation. In this section, we analyze the different mathematical constraints of the EFIE and MFIE integral operators on the the surface electric currents and explain why basis functions which do not impose normal continuity between elements can be used to discretize the MFIE.

2.1. EFIE and MFIE formulations

For electromagnetic scattering from PEC objects, the electric field integral equation (EFIE) can be achieved from the imposition of the electric field boundary condition on the surface of a PEC body. If the surface of the PEC objects under analysis is S and the incident and scattered electric fields are respectively of $\mathbf{E}^i(\mathbf{r})$ and $\mathbf{E}^s(\mathbf{r})$, then the EFIE can be written as

$$\mathbf{E}^s(\mathbf{r})|_{\tan} = -\mathbf{E}^i(\mathbf{r})|_{\tan} \tag{1}$$

By the use of the Green's function in free space $G(\mathbf{r}, \mathbf{r}')$ and the surface equivalent electric current $\mathbf{J}_s(\mathbf{r})$, the scattered fields $\mathbf{E}^s(\mathbf{r})$ can be further expressed as

$$\mathbf{E}^s(\mathbf{r}) = \iint_S [j\omega\mu \mathbf{J}_s(\mathbf{r}')G(\mathbf{r}, \mathbf{r}') - \frac{j}{\omega\varepsilon}(\nabla' \cdot \mathbf{J}_s(\mathbf{r}'))\nabla' G(\mathbf{r}, \mathbf{r}')]ds' \tag{2}$$

In (2), ω is the working angular frequency and μ and ε are respectively the permittivity and permeability of free space.

Similarly, the MFIE arises from the imposition of the magnetic field boundary condition over the boundary surface S around a PEC body. It can be written as

$$\mathbf{J}_s(\mathbf{r}) = \mathbf{n}(\mathbf{r}) \times \mathbf{H}(\mathbf{r}) \tag{3}$$

which can be further expressed as

$$\mathbf{n}(\mathbf{r}) \times \mathbf{H}^i(\mathbf{r}) = \frac{1}{2}\mathbf{J}_s(\mathbf{r}) - \mathbf{n}(\mathbf{r}) \times p.v. \iint_S \mathbf{J}_s(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}')ds' \tag{4}$$

by expressing the scattered magnetic fields $\mathbf{H}^s(\mathbf{r})$ through the use of the Green's function and the surface currents $\mathbf{J}_s(\mathbf{r})$. In (3), $\mathbf{H}(\mathbf{r})$ is the total magnetic field and $\mathbf{n}(\mathbf{r})$ is the unit normal vector of the surface S. In (4), $\mathbf{H}^i(\mathbf{r})$ is the incident magnetic field.

2.2. Choice of basis functions for EFIE and MFIE

Although the same equivalent surface electric current $\mathbf{J}_s(\mathbf{r})$ appears in the EFIE and the MFIE, basis functions used to discretize the corresponding surface equivalent electric currents $\mathbf{J}_s(\mathbf{r})$ may be different. This has been observed in many literatures. In [1], it is proposed that the normal component of the equivalent electric currents $\mathbf{J}_s(\mathbf{r})$ should be continuous across the edges between neighboring elements and the divergence-conforming RWG basis functions were used to expand the equivalent electric currents $\mathbf{J}_s(\mathbf{r})$ in the EFIE. In [3,7], the same basis functions are also used to the MFIE. However, it is shown in [12] that the monopolar RWG basis functions as well as the curl-conforming basis functions in [15] can also be used to expand $\mathbf{J}_s(\mathbf{r})$ in the MFIE. The monopolar RWG basis function is only defined on a single triangular element and both the normal and the tangential component of the surface currents are not continuous between neighboring elements. The curl-conforming basis functions only guarantee the continuity of the tangential component of the surface currents. These two kinds of basis functions do not guarantee the continuity of the normal component of the equivalent electric currents. In fact, the RCS results of the MFIE resulted from the use of these two kinds of basis functions are even much better than those using the RWG basis functions. However, although the numerical results of the MFIE using none divergence-conforming basis functions are shown in [12,15], no any explanation has been given to why these kinds of basis functions can be used to expand the surface currents $\mathbf{J}_s(\mathbf{r})$ in the MFIE.

It can be seen from the EFIE in (2) that both the equivalent electric currents $\mathbf{J}_s(\mathbf{r})$ and its divergence $\nabla \cdot \mathbf{J}_s(\mathbf{r})$, i.e. the equivalent electric charges, contribute to the scattered electric fields. To correctly model $\mathbf{J}_s(\mathbf{r})$, the basis functions used to expand $\mathbf{J}_s(\mathbf{r})$ should make $\nabla \cdot \mathbf{J}_s(\mathbf{r})$ be an integrable function everywhere on the surface in analysis. When the surface of a PEC object is discretized with triangular elements, any polynomial functions defined on elements will make $\mathbf{J}_s(\mathbf{r})$ be a finite and continuous function in the corresponding domain. Besides, they usually make $\nabla \cdot \mathbf{J}_s(\mathbf{r})$ be a finite value in the inner part of a single element. However, on the three edges of a triangular element, arbitrarily selected polynomial functions could not guarantee the continuity of the normal component of $\mathbf{J}_s(\mathbf{r})$. The discontinuity of $\mathbf{J}_s(\mathbf{r})$ on the boundaries of an element will make $\nabla \cdot \mathbf{J}_s(\mathbf{r})$ be a singular function on these boundaries. Because the equivalent surface currents are associated with the equivalent surface charges through the continuous equation, it can also be explained as the radiation of line charges on boundaries of elements. Therefore, to calculate the scattered electric fields correctly, both the contribution of the electric currents $\mathbf{J}_s(\mathbf{r})$ and its divergence $\nabla \cdot \mathbf{J}_s(\mathbf{r})$ should be modeled correctly. The normal continuity condition of the $\mathbf{J}_s(\mathbf{r})$ makes the choice of the basis function limited in a divergence-conforming function space. If other basis functions which can not guarantee the normal continuity condition are used to expand the electric currents in the EFIE, the contribution of the accumulated electric charges to the scattered electric fields will results in error RCS values. This has been observed in [12], but no any further explanation was given in that paper. In [10,11], special techniques have been developed to deal with the contribution of the accumulation of the electric charges on the boundaries of elements.

Different from the case of the EFIE, for the MFIE in (4), it can be seen clearly that only the surface currents $\mathbf{J}_s(\mathbf{r})$ contribute to the calculation of the scattered magnetic fields. There is no contribution from $\nabla \cdot \mathbf{J}_s(\mathbf{r})$, i.e., the equivalent charges. Therefore, even the normal component of $\mathbf{J}_s(\mathbf{r})$ is not continuous across element boundaries, the accumulated line charges on the boundaries of an element will not contribute to the scattered magnetic fields. If the curl-conforming basis functions are used to expand the surface