Engineering Analysis with Boundary Elements 65 (2016) 72–78

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Complex variable moving Kriging interpolation for boundary meshless method

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ARTICLE INFO

Article history:
Received 29 October 2015
Received in revised form
7 December 2015
Accepted 5 January 2016
Available online 22 January 2016

Keywords:
Boundary node method
Moving Kriging interpolation
Complex variable
Potential problem

ABSTRACT

In this paper, we proposed the complex variable moving Kriging interpolation (CVMKI) to approximate functions on two-dimensional (2D) boundaries. The CVMKI is based on complex variable theory and the moving Kriging interpolation (MKI). It requires no curvilinear coordinate, and can construct shape functions possessing Kronecker delta function property and partition of unity property. Further, the complex variable boundary node method (CVBNM) is proposed for potential problems based on CVMKI and boundary integration equation (BIE). CVBNM is an efficient and accurate method that can directly impose the boundary conditions. Three 2D example problems are presented to verify the accuracy and efficiency of CVBNM.

1. Introduction

The meshfree methods have attracted growing attention in numerical simulations because of its greater flexibility and higher precision than the conventional methods such as finite element method (FEM) and boundary element method (BEM). A meshfree method requires no cell for interpolation, and constructs the shape function entirely based on scattered nodes. 'Domain' type meshfree methods such as smooth particle hydrodynamics (SPH) [1] method, reproducing kernel particle method (RKPM) [2], the element free Galerkin (EFG) method [3], the point interpolation method (PIM) [4,5], the meshless local Petrov–Galerkin (MLPG) [6–10] method, the complex variable element free Galerkin (CVEFG) [11–15] method, the improved complex variable element-free Galerkin (ICVEFG) method [16], the complex variable meshless local Petrov-Galerkin (CVMLPG) method [17–19], the complex variable reproducing kernel particle method (CVRKPM) [20–23], the complex variable meshless manifold method (CVMMM) [24], have been proposed and applied in many engineering problems.

The idea of meshfree has also been introduced in 'boundary' type method. Mukherjee et al. [25–27] have proposed the boundary node method (BNM) based on the moving least square (MLS) method [28] and the boundary integral equation (BIE). Atluri et al. [29,30] have developed an important meshfree method, called the local boundary integral equation (LBIE) method for nonlinear problems and non-homogeneous domains. Zhang et al. [31–33] have proposed the hybrid boundary node method (HdBNM) that only requires nodes distributed on the boundary without considering any cell for interpolation or for integration. Based on different approximation methods and BIE, Cheng et al. have proposed the boundary element-free method (BEFM) [34–38], the improved boundary element-free method [39,40], the complex variable boundary element-free method [41] and the reproducing kernel particle boundary element-free method (RPKBNM) [42]. Dai et al. [43] have proposed the moving Kriging interpolation-based boundary node method (MKIBNM) by combing moving Kriging interpolation (MKI) with BIE for potential problems. Zhang et al. [44–46] have proposed the boundary face method (BFM) for many engineering problems.

Approximation methods such as SPH, MLS, CVMLS and MKI play an important role in constructing the shape function in meshless methods. MLS is one of the most widely used approximation methods, because it can form the approximation function with high precision. Based on MLS and complex variable theory, Cheng et al. [11,13,14,47] have proposed the complex variable moving least squares (CVMLS) approximation. With the application of the complex variable theory, the trial function of a 2D domain problem can be formed with a one-dimensional (1D) basis function. Therefore, the unknown coefficients in the trial function of the CVMLS approximation are less than that of the MLS approximation. Thus, the computational efficiency and stability increases. Besides, the complex variable theory can also be applied in boundary type meshless method.

The 2D boundary problems are essentially 1D problems. In the boundary type meshless methods, if we try to construct shape functions directly using Cartesian coordinates, the coefficient

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http://dx.doi.org/10.1016/j.enganabound.2016.01.003
0955-7997/

matrix will be singular or ill-conditioned. Thus, the curvilinear coordinates [25,48,49] are introduced to construct shape functions. However, it is burdensome and time-consuming to obtain the curvilinear coordinates for some problems, such as complicated boundary problems and moving boundary problems. The complex variables can be used to construct shape functions for boundary problems instead of the curvilinear coordinates, because it enables the trial function of a 2D problem to be formed with 1D basis functions and avoids the singular or ill-conditioned coefficient matrix. Besides, the complex variable can be obtained directly and easily from the Cartesian coordinates. Based on the CVMLS approximation, Cheng et al. [41] have proposed the complex variable boundary element-free method (CVBEFM) for 2D elastodynamic problems.

According to the above methods, CVMLS brings efficiency and stability to domain type meshless methods and convenience to boundary type meshless methods. However, CVMLS inherits the drawback of MLS that the shape functions do not satisfy the Kronecker delta property. Thus, the essential boundary conditions need to be imposed by techniques such as Lagrange multipliers method [3], direct collocation methods [50], penalty methods [51] and modified variation principles [50,52].

In some meshless methods, the moving Kriging interpolation (MKI) has been used to construct shape functions. The approximation precision of MKI is higher than that of MLS. Besides, the shape functions constructed by MKI possess Kronecker delta property. Then, the essential boundary condition can be imposed directly and easily. Gu [53] has firstly introduced MKI and has successfully demonstrated the effectiveness of MKI in solving 2D steady-state heat conduction problems. Based on MKI, Dai et al. have proposed the moving Kriging interpolation-based boundary node method (MKIBNM) [43] and the improved meshless local Petrov–Galerkin (MLPG) [7–9] method, and Yimnak et al. [54] have developed the local integral equation formulation for solving coupled nonlinear reaction-diffusion equations.

In this paper, we proposed the complex variable moving Kriging interpolation (CVMKI) based on the complex variable theory and MKI. CVMKI only requires Cartesian coordinates of the boundary nodes to construct shape functions possessing Kronecker delta property. Further, based on CVMKI and BIE, we proposed the complex variable boundary node method (CVBNM), which requires no curvilinear coordinate, can directly impose the boundary conditions and has high efficiency.

2. Complex variable moving Kriging interpolation (CVMKI) on 2D boundary

In CVMKI, the complex variable $z = x + iy$ is used instead of the curvilinear coordinate s to represent the position of points and approximate boundary variables (Fig. 1).

Similar to the MKI approximation [7,53,55], the local approximation of the function $u(z)$ on the boundary can be defined by

$$u^h(z) = \sum_{j=1}^{m} p_j(z) a_j + \gamma(z) = \boldsymbol{p}(z)\boldsymbol{a} + \gamma(z) \tag{1}$$

where $\boldsymbol{p}(z) = [p_1(z), p_2(z), \cdots, p_m(z)]$, $p_j(z) = z^{j-1}$ $(j = 1, 2, \cdots, m)$ are monomial basis functions, m is the number of terms in basis. $\boldsymbol{a}^T = [a_1, a_2, \cdots, a_m]$, a_j $(j = 1, 2, \cdots, m)$ are the coefficients of monomial basis functions, and the random process $\gamma(z)$ is assumed to have the following properties.

$$\begin{cases} E[\gamma(z)] = 0 \\ D[\gamma(z)] = \sigma^2 \\ Cov\{\gamma(z_i),\ \gamma(z_j)\} = \sigma^2 R(z_i,\ z_j) \end{cases} \tag{2}$$

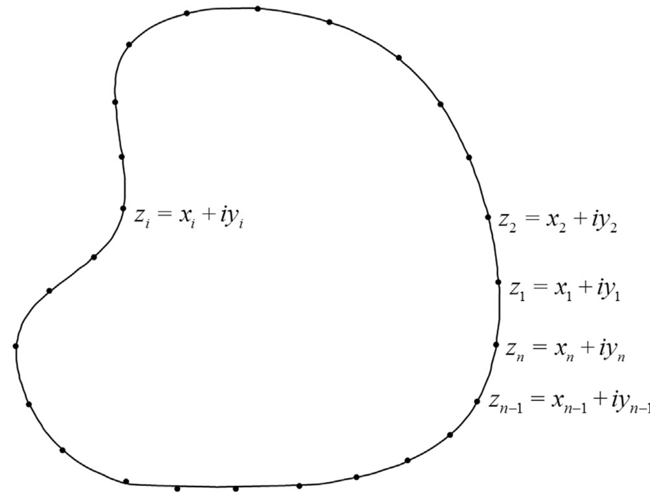

Fig. 1. Complex variable on the boundary.

where, σ^2 is the variance, $R(z_i, z_j)$ is the correlation function between node z_i and node z_j, and Gaussian function is chosen as the correlation function in this work.

$$R(z_i,\ z_j) = \exp\left(-\frac{\omega r_{ij}^2}{d_m^2}\right) \tag{3}$$

where $r_{ij} = |z_i - z_j|$ is the norm of $z_i - z_j$ and also the distance between z_i and z_j, d_m is the minimum distance between any two nodes on the sub-boundary, $\omega > 0$ is a correlation parameter and $\omega = 0.03 - 0.2$ is recommended [7].

Substituting the given set of boundary nodes $\{z_1, z_2, \cdots, z_n\}$ and the corresponding function values $\boldsymbol{U} = [u_1, u_2, \cdots, u_n]^T$ into Eq. (1) yields.

$$\boldsymbol{U} = \boldsymbol{Pa} + \Upsilon \tag{4}$$

where, $\boldsymbol{P}$ is the $n \times m$ matrix that has basis function values at the given set of nodes.

$$\boldsymbol{P} = \begin{bmatrix} p_1(z_1) & p_2(z_1) & \cdots & p_m(z_1) \\ p_1(z_2) & p_2(z_2) & \cdots & p_m(z_2) \\ \vdots & \vdots & \ddots & \vdots \\ p_1(z_n) & p_2(z_n) & \cdots & p_m(z_n) \end{bmatrix} \tag{5}$$

$\boldsymbol{\Upsilon}$ is the $n \times 1$ vector of the error between regression model and real process.

$$\boldsymbol{\Upsilon}^T = [\gamma_1, \gamma_2, \cdots, \gamma_n] \tag{6}$$

At any $z \in \Gamma_s$, $u(z)$ can also be estimated by the linear predictor.

$$\hat{u}(z) = \boldsymbol{\Phi}(z)\boldsymbol{U} = [\phi_1, \phi_2, \cdots, \phi_n] \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \tag{7}$$

The error between $\hat{u}(z)$ and $u^h(z)$ is

$$\begin{aligned} \hat{u}(z) - u^h(z) &= \boldsymbol{\Phi}(z)\boldsymbol{U} - \boldsymbol{u}^h(z) \\ &= \boldsymbol{\Phi}(z)(\boldsymbol{Pa} + \Upsilon) - (\boldsymbol{p}(z)\boldsymbol{a} + \gamma(z)) \\ &= \boldsymbol{\Phi}(z)\boldsymbol{\Upsilon} - \gamma(z) + (\Phi(z)P - p(z))\boldsymbol{a} \end{aligned} \tag{8}$$

To ensure the unbiased predictor, Eq. (9) has to be satisfied.

$$\boldsymbol{\Phi}(z)\boldsymbol{P} - \boldsymbol{p}(z) = \boldsymbol{0} \tag{9}$$

Substituting Eq. (9) into Eq. (8), we can obtain

$$\hat{u}(z) - u^h(z) = \boldsymbol{\Phi}(z)\boldsymbol{\Upsilon} - \gamma(z) \tag{10}$$

Assuming $\hat{u}(z)$ as random, we can compute the mean squared error (MSE) of $\hat{u}(z)$. Thus, the best linear unbiased predictor (BLUP)