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Effect of surface slip on the relative motion and collision efficiency of slippery spherical particles

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ABSTRACT

The motion of a pair of spherical particles suspended in a viscous fluid is considered under conditions of Stokes flow. The particle surfaces allow the fluid to slip according to the Navier–Maxwell–Basset law. Batchelor and Green's mobility functions determining the relative particle motion during interception are computed with high accuracy using a boundary-integral method, and their dependence on the slip coefficient is discussed. The numerical results confirm that particle collision in the presence of surface slip occurs with a finite impact velocity. As the slip coefficient decreases, and thereby the particle surfaces become increasingly slippery, the collision efficiency in uniaxial elongational and simple shear flow increases monotonically from zero for no-slip surfaces to a finite limit for perfectly slippery surfaces.

1. Introduction

Luo and Pozrikidis [1] developed a boundary-integral method for computing the interception of two spherical particles in linear Stokes flow, where the fluid is allowed to slip over the particle surfaces according to the Navier–Maxwell–Basset law [2]. The physical objective was to provide insight into the hydrodynamics of suspensions at small scales where the no-slip (stick) boundary condition fails and the fluid appears to slip with a velocity that is proportional to the local shearing component of the traction. If the ambient fluid is a rarefied gas, the slip coefficient can be related rigorously to the mean free path by the Maxwell relation in terms of the Knudsen number and the tangential momentum accommodation coefficient (TMAC) (e.g., [3,4]). Macroscopic (apparent) slip arises in the flow past nonidealized particles with irregular surfaces due to natural boundary roughness. Flow over a hydrophobic surface appears to slip with a slip length on the order of micrometers.

Luo and Pozrikidis [1] found that two particles intercepting in simple shear flow collide when the initial lateral particle offset is sufficiently small and the slip coefficient is sufficiently low. In the absence of slip, the particles are not able to collide due to strong lubrication forces developing between the particle surfaces in close proximity. Instead, the particles roll over and bypass each other, or else engage in a perpetual orbiting motion. In this paper, the hydrodynamic interaction of two particles with slip surfaces is further considered.

Following the landmark analysis of Batchelor and Green [5] for spherical solid particles and liquid drops, we express the velocity of one slippery spherical particle labeled A with radius a relative to the velocity of a second slippery particle labeled B with radius δa in an infinite linear flow as

$$\mathbf{V}_A-\mathbf{V}_B=\omega^\infty\times\mathbf{r}+\mathbf{E}^\infty\cdot\mathbf{r}-\left(\frac{\mathcal{A}(s)}{r^2}\mathbf{r}\otimes\mathbf{r}+\frac{\mathcal{B}(s)}{r^2}(r^2\mathbf{I}-\mathbf{r}\otimes\mathbf{r})\right)\cdot\mathbf{E}^\infty\cdot\mathbf{r},\tag{1.1}$$

where $\mathbf{r}$ is the relative position of the particle centers, $s=|\mathbf{r}|/a_m$ is a scaled dimensionless distance, $a_m=\frac{1}{2}(1+\delta)\,a$ is the mean particle radius, ω^∞ is half the vorticity vector of the unperturbed linear flow, and $\mathbf{E}^\infty$ is the rate-of-deformation tensor of the unperturbed linear flow. The dimensionless functions $\mathcal{A}(s)$ and $\mathcal{B}(s)$ are the axial and transverse relative mobility coefficients.

Our first objective in this paper is to present accurate data on these functions over a broad range of particle separations, well into the regime of small gaps, and to discuss their dependence on the slip coefficient. The results will confirm that surface slip allows the particles to collide at a finite time with a nonzero impact velocity.

When a particle suspension is stirred, particles intercept one another and agglomerate permanently or temporarily during collision. Flow-induced agglomeration renders the suspended phase hydrodynamically unstable. The collision efficiency is the ratio of the rate of particle collision to that occurring when the particles follow the streamlines of the unperturbed flow (e.g., [6]). Because particles with no-slip surfaces are unable to collide at a finite time due to

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strong lubrication forces developing at small separations, the collision efficiency is zero. In contrast, perfectly spherical bubbles and drops can touch at a finite time, yielding a nonzero collision efficiency (e.g., [7]). Interfacial deformation at small separations prevents the interfaces from touching. However, if the thickness of the film separating the interfaces of two drops or bubbles is sufficiently small, attractive intermolecular forces cause adhesion or film rupture and allow for coalescence below a certain threshold.

Our second objective in this paper is to investigate the effect of the particle surface slip of the collision efficiency of a monodisperse suspension. The results will show that, as the slip coefficient decreases and thereby the surfaces become increasingly slippery, the collision efficiency increases for both extensional and simple shear flow.

2. Interception of two spherical particles

We consider viscous flow past two suspended spherical particles in an effectively infinite fluid. Far from the particles, the velocity field takes the linear form

$$\mathbf{U}^{\infty}(\mathbf{X}) = \mathbf{L}^{\mathrm{T}} \cdot \mathbf{X}, \tag{2.1}$$

where $\mathbf{L}$ is the velocity gradient tensor with components $L_{ij} = \partial U_j / \partial X_i$, the superscript T denotes the matrix transpose, and $\mathbf{X} = (X, Y, Z)$ is the position in laboratory-fixed coordinates, as shown in Fig. 1. The presence or motion of the particles generates a disturbance flow, denoted by the superscript D, that may be added to the incident linear flow to yield the total physical flow with velocity $\mathbf{U} = \mathbf{U}^{\infty} + \mathbf{U}^{\mathrm{D}}$.

The no-penetration and slip boundary conditions are assumed over the particle surfaces, requiring that the surface velocity is given by

$$\mathbf{U} = \mathbf{V}^{(i)} + \Omega^{(i)} \times (\mathbf{X} - \mathbf{X}_c^{(i)}) + \mathbf{U}^{\mathrm{S}}, \tag{2.2}$$

where $\mathbf{V}^{(i)}$ is the velocity of translation of the ith particle center $\mathbf{X}_c^{(i)}$ for $i = 1, 2$, and $\Omega^{(i)}$ is the angular velocity of rotation about $\mathbf{X}_c^{(i)}$. The slip velocity is given by the Navier–Maxwell–Basset relation

$$\mathbf{U}^{\mathrm{S}} = \frac{L}{\mu\beta} \mathbf{N} \times \mathbf{F} \times \mathbf{N} = \frac{\lambda}{\mu} \mathbf{N} \times \mathbf{F} \times \mathbf{N}, \tag{2.3}$$

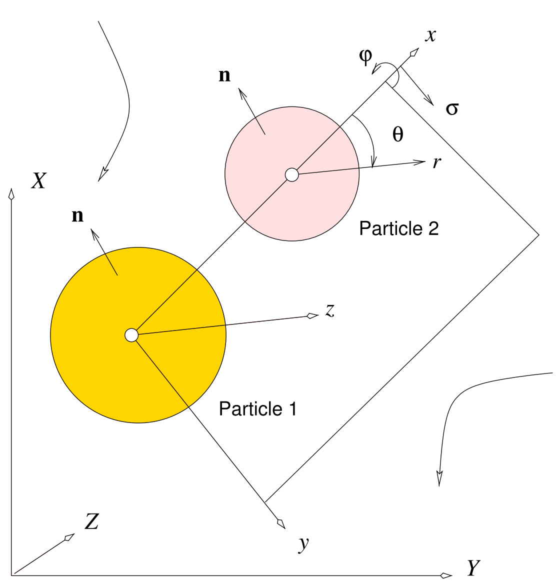

Fig. 1. Two spherical particles with arbitrary radii intercept in a linear flow; (X, Y, Z) are the global coordinates fixed at the laboratory frame, (x, y, z) are particle doublet coordinates, (r, θ, φ) are corresponding spherical polar coordinates, and (x, σ, φ) are cylindrical polar coordinates attached to one particle.

where μ is the fluid viscosity, $\mathbf{F} \equiv \mathbf{\Sigma} \cdot \mathbf{N}$ is the surface traction, $\mathbf{\Sigma}$ is the stress tensor, $\mathbf{N}$ is the unit normal vector pointing into the fluid, L is a chosen characteristic length scale, β is the dimensionless Basset slip coefficient ranging from zero in the case of vanishing shear stress and perfect slip to infinity in the case of no slip, and $\lambda = L/\beta$ is the particle surface slip length. As β tends to zero, the shear stress vanishes to allow for a finite slip velocity. Thus, particles with perfectly slippery surfaces can be regarded as gas bubbles enclosed by free surfaces where the zero-shear-stress condition applies.

To recast the problem into a canonical flow, we introduce a new coordinate system, (x, y, z), with origin at the center of the first sphere, $\mathbf{X}_c^{(1)}$. The x-axis passes through the centers of the two particles, while the y and z axes point into two orthogonal but otherwise unspecified directions. Thus, the definition of the particle coordinate system affords one inconsequential degree of freedom. The position vector and the velocity transform according to the equations

$$\mathbf{X} = \mathbf{X}_c^{(1)} + \mathbf{A} \cdot \mathbf{x}, \quad \mathbf{u} = \mathbf{A}^{\mathrm{T}} \cdot \mathbf{U}, \tag{2.4}$$

where $\mathbf{A}$ is an orthogonal transformation matrix hosting in its columns the direction cosines of the unit vectors along the x, y, and z axes,

$$\mathbf{A} = \begin{bmatrix} (\mathbf{e}_x)_X & (\mathbf{e}_y)_X & (\mathbf{e}_z)_X \\ (\mathbf{e}_x)_Y & (\mathbf{e}_y)_Y & (\mathbf{e}_z)_Y \\ (\mathbf{e}_x)_Z & (\mathbf{e}_y)_Z & (\mathbf{e}_z)_Z \end{bmatrix} \equiv \begin{bmatrix} A_{Xx} & A_{Xy} & A_{Xz} \\ A_{Yx} & A_{Yy} & A_{Yz} \\ A_{Zx} & A_{Zy} & A_{Zz} \end{bmatrix} \tag{2.5}$$

with

$$\mathbf{e}_x = \frac{\mathbf{X}_c^{(2)} - \mathbf{X}_c^{(1)}}{\left|\mathbf{X}_c^{(2)} - \mathbf{X}_c^{(1)}\right|}, \tag{2.6}$$

$\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}$, where δ_{ij} is Kronecker's delta. Applying the velocity transformation rules for the incident linear flow, we find that

$$\mathbf{u}^{\infty}(\mathbf{x}) = \mathbf{A}^{\mathrm{T}} \cdot \mathbf{L}^{\mathrm{T}} \cdot \mathbf{X} = \mathbf{A}^{\mathrm{T}} \cdot \mathbf{L}^{\mathrm{T}} \cdot (\mathbf{X}_c^{(1)} + \mathbf{A} \cdot \mathbf{x}), \tag{2.7}$$

which can be rewritten as

$$\mathbf{u}^{\infty}(\mathbf{x}) = \mathbf{v}^{\infty} + \mathbf{M}^{\mathrm{T}} \cdot \mathbf{x}, \tag{2.8}$$

where $\mathbf{v}^{\infty} = \mathbf{A}^{\mathrm{T}} \cdot \mathbf{L}^{\mathrm{T}} \cdot \mathbf{X}_c^{(1)}$ and $\mathbf{M} = \mathbf{A}^{\mathrm{T}} \cdot \mathbf{L} \cdot \mathbf{A}$. The orthogonality of the matrix $\mathbf{A}$ ensures that the trace of the velocity gradient tensor in the particle frame, $\mathbf{M}$, is zero.

Luo and Pozrikidis [1] developed a boundary-integral method for solving the governing equations in the particle-doublet frame and simultaneously computing the particle translational and angular velocities. At any instant, the flow is computed in a frame of reference with origin at the center of one particle using a cylindrical polar coordinate system whose axis of revolution passes through the center of the second particle. Taking advantage of the axial symmetry of the boundaries of the flow in the particle coordinates, the problem is formulated as a system of integral equations for the zeroth, first, and second Fourier coefficients of the boundary traction with respect to the azimuthal angle, φ, measured around the axis connecting the particle centers, as shown in Fig. 1. The force and the torque exerted on each particle are determined by the zeroth and first Fourier coefficients, while the stresslet is determined by the zeroth, first, and second Fourier coefficients.

The integral equations were solved with high accuracy using a boundary-element method featuring adaptive element distribution and automatic time-step adjustment according to the interparticle gap. The semicircular particle contours in the $\varphi = 0$ azimuthal plane were divided into circular elements and the Fourier coefficients were approximated with constant functions over each element. For best accuracy, the elements were concentrated near the axis of symmetry so that their size increases geometrically