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Buckling analysis of functionally graded thin plate with in-plane material inhomogeneity

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A B S T R A C T

Buckling analysis of functionally graded material (FGM) thin plates with in-plane material inhomogeneity is investigated based on radial basis functions associated with collocation method. No background mesh is required in the discretization and solution which makes it a truly meshfree method. Two independent problems raised in the buckling analysis are studied according to the procedure. First, radial basis collocation method (RBCM) is employed to yield the non-uniform pre-buckling stresses by solving a 2D plane stress problem. Afterwards, based on Kirchhoff assumption and employing the predetermined non-uniform pre-buckling stresses, Hermite radial basis function collocation method (HRBCM) is proposed to study the buckling loads of FGM thin plates with in-plane material inhomogeneity. Compared to an over-determined system resulting from the conventional RBCM, HRBCM introducing more degrees of freedom on the boundary nodes can lead to a determined system for the eigenvalue problem. Convergence and comparisons studies with analytical solutions demonstrate that the proposed method possesses high accuracy and exponential convergence. Numerical examples illustrate that the material inhomogeneity has considerable effects on the buckling loads and mode shapes of thin plates. As a result, material inhomogeneity can be exploited to optimize the in-plane stress distribution and prevent the buckling of thin plates.

1. Introduction

Traditional composite materials whose constituents are distributed either uniformly or randomly suffer the weakness of interfaces between layers which may lead to delamination or even debonding. This weakness brought in the new idea of formatting the material properties varying in a continuous manner according to the demanding performances and this resulted in the generation of functionally graded materials (FGMs). Mechanical properties were the first study interest of FGMs structures. Kashtalyan [1] investigated the bending of functionally graded rectangular plates based on a three-dimensional elasticity solution. Further, Zhong and Shang [2] proposed closed-form solutions for some specific variations of material modulus to examine the bending analysis in three-dimensional functionally graded plates. Vibration analysis for FGMs was studied based on analytical solutions according to Fourier series expansion [3] and some numerical methods such as meshfree radial point interpolation method [4] etc. By utilizing sinh function as an non-linear co-ordinate transformation to deal with the nearly-singular integrals in Boundary Element Method (BEM), Gu etc. [5] studied the stress distributions for the thin multilayered coating system. In addition, thermal analysis [6,7] and fracture mechanics [8,9] were also extensively explored for FGM structures.

FGMs can be utilized for optimizing the material's properties in microstructure which can also improve the buckling behavior of a structure. Feldman and Aboudi [10] studied the elastic bifurcational buckling of functionally graded plates under in-plane compressive loading and gave an optimal spatial distribution of the reinforcement phase. Chen and Liew [11] investigated the buckling behavior of FGM Mindlin's plates subjected to pin loads, partial uniform loads and parabolic loads based on meshfree radial basis functions method. Bodaghi and Saidi [12] proposed an analytical solution for the buckling of a rectangular FGM thin plate subjected to non-uniformly distributed in-plane loading acting on two opposite simply supported edges. By employing a micromechanics-based model, Shariyat and Asemi [13] presented a non-linear three-dimensional energy-based elasticity analysis for buckling investigation of functionally graded plates subjected to non-uniform in-plane compressions. Furthermore, the mechanical and thermal buckling analysis of functionally graded ceramic-metal plates was presented by Zhao [14] etc., using the first-order shear deformation plate theory associated with the element-free kp-Ritz method. Buckling of functionally graded materials with inhomogenity along thickness direction have been well studied as shown in the above mentioned representative work, in which the stresses are uniformly

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distributed in the plane. However, FGMs with in plane inhomogeneity which have attracted great attentions because of their extensive applications [15,16] are still short of investigation, in which the pre-buckling stresses are distributed non-uniformly and should be generated in advance of the buckling analysis.

In the past decades, radial basis collocation method (RBCM) has become one of the primary and outstanding methodology for solving partial differential equations (PDEs) [17–23], which also found its great use in the applications of FGMs because of its easy discretization in irregular domains and possessing spectral accuracy for solving PDEs with variable coefficients. Based on radial basis functions in conjunction with collocation methods, Ferreira [24] studied the static deformations and Roque etc. [25] studied the free vibration of functionally graded plates by a third-order shear deformation theory and a refined theory. Taking RBFs as approximation, Gilhooley etc. [26] used meshless local Petrov–Galerkin (MLPG) method and Dai etc. [4] used meshless radial point interpolation method to analyze two-dimensional (2D) static and dynamic deformations of functionally graded materials (FGMs). Further, Mojdhi [27] introduced MLPG for static and dynamic analysis of thick functionally graded plates in three dimension. Employing radial basis collocation method associated with pseudospectral methods, RBFs were also extended to solve the linear transient response of functionally graded plates and shells using first-order shear deformation theory [28].

When dealing with thin plate problems, RBFs method is also a great candidate for solving eigenproblems [29]. However, the conventional RBCM leads to an over-determined system for the eigenproblems in thin plate problem because there are two boundaries conditions corresponding to one boundary point. Therefore, Hermite radial basis collocation method (HRBCM) [30–33] which can result in a determined system was proposed by Chu etc. [34] for the free vibration of thin FGM plates with in-plane material inhomogeneity. In this paper, a solution technique based on RBFs and collocation method is presented to analyze the buckling behavior of FGM thin plates with in-plane material inhomogeneity. Two steps need to be executed for the buckling analysis. First, radial basis collocation method (RBCM) is adopted to procure the non-uniform pre-buckling stresses in a corresponding 2D plane stress problem. Second, Hermite radial basis collocation method (HRBCM) is proposed for the eigenproblem analysis of thin plates by introducing the predetermined non-uniform pre-buckling stresses and invoking the Kirchhoff assumption.

This paper is organized as follows. Governing equations of the FGM thin plates with in-plane material inhomogeneity is formulated in Section 2. In Section 3, RBFs are introduced, and then RBCM for plane stress problem and HRBCM for eigenproblem analysis are presented. Several numerical examples are given in Section 4 to examine the accuracy and convergence of the proposed method for the buckling problem, and the effects of material inhomogeneity on the buckling loads and mode shapes of thin plates are also summarized. Finally, the concluding remarks are given in Section 5.

2. Governing equations

2.1. Basic equations of FGM thin plate

Consider a rectangular FGM thin plate of uniform thickness h and its infinitesimal element is shown in Fig. 1. The material properties vary in the plane of the plate. The $xoy(z=0)$ plane coincides with the midplane of the thin plate. $q=q(x,y)$ is the transverse load per unit area. The flexural rigidity of the plate is $D(x,y)=Eh^3/12(1-\nu^2)$, where $E=E(x,y)$ is the Young's modulus and ν is the Poisson's ratio. Based on Kirchhoff thin plate assumption, the strains across the plate thickness at a distance z away from the middle plane are

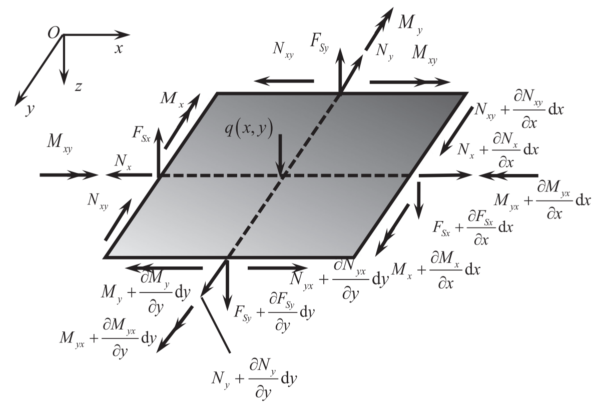

Fig. 1. Infinitesimal element of FGM thin plate with in-plane material inhomogeneity and sign conventions of inner forces.

Table 1
First buckling loads of homogeneous plates.

Boundary conditions	Load modes	Analytical solution	Numerical solution		
			7 × 7	9 × 9	13 × 13
SSSS	Perpendicular	39.4784	39.1579	39.3446	39.4109
	Parallel	39.4784	39.1579	39.3446	39.4109
	Bidirectional	19.7392	19.5787	19.6490	19.6845
	Shear	92.1821	93.7968	92.7037	92.3312
CCCC	Perpendicular	99.3869	99.7426	99.5543	99.4597
	Parallel	99.3869	99.8426	99.5943	99.5097
	Bidirectional	52.6050	52.4589	52.5183	52.5569
	Shear	145.1819	147.0816	145.6708	145.2794

$$\varepsilon_x=\varepsilon_{x0}+z\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y=\varepsilon_{y0}+z\frac{\partial^2 w}{\partial y^2}$$

$$\varepsilon_{xy}=\varepsilon_{xy0}+2z\frac{\partial^2 w}{\partial x\partial y} \tag{1}$$

where ε_{x0}, ε_{y0} and ε_{xy0} are the normal strains and the shear strain at the middle plane of the plate, w is the deflection of the plate. The generalized Hooke's law gives the stresses as

$$\sigma_x=\frac{E(x,y)}{1-\nu^2}(\varepsilon_x+\nu\varepsilon_y)$$

$$\sigma_y=\frac{E(x,y)}{1-\nu^2}(\varepsilon_y+\nu\varepsilon_x)$$

$$\sigma_{xy}=\frac{E(x,y)}{1+\nu}\varepsilon_{xy} \tag{2}$$

The moment per unit length and the in-plane forces per unit length of the thin plate are given as

$$M_x=\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_x dz, M_y=\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_y dz, M_{xy}=\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_{xy} dz$$

$$N_x=\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz, N_y=\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y dz, N_{xy}=\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} dz \tag{3}$$

The general equilibrium equations for the thin plate is

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+q(x,y)=0 \tag{4}$$

Substituting (1)–(3) into (4), after some mathematical manipulations, the governing equation of FGM thin plate with in-plane