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A comparative analysis of local meshless formulation for multi-asset option models

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ABSTRACT

A local meshless radial basis function collocation differential quadrature (LMRBFCDQ) is proposed for the numerical solution of a single and multi-asset option pricing PDE models arising in computational finance. Spatial discretization is performed by both local and a standard global meshless collocation procedures coupled with a set of different time integrators based on the forward Euler difference formula (FEDF), the fully Implicit method (FIM), the Crank–Nicolson method (CNM), the explicit Runge–Kutta method of order two (ERK2), the Crank–Nicolson Runge–Kutta method of order two (CNRK2), the fully Implicit Runge–Kutta method of order two (IRK2), the Runge–Kutta method of order four (RK4), the Embedded Runge–Kutta method (RK23). Operator splitting techniques like the ordinary operator splitting (OOS), the Lie–Trotter splitting, the additive splitting and the Strang splitting are also tested for time integration. The proposed hybrid schemes are the amalgamation of the meshless differential quadrature procedure and the finite difference approximations. Different types of radial basis functions (RBFs) i.e. the multiquadric (MQ), the inverse quadric (IQ) and the Gaussian (GA) are utilized for the spatial discretization of the PDE models. Numerical analysis of a range of computational finance related models are shown to demonstrate accuracy, efficiency and ease of implementation of the proposed meshless-finite difference procedure.

1. Introduction

In recent years, their is a growing interest to study financial derivatives through the use of mathematical modelling and simulation. Financial derivatives are the instruments when used sagaciously can maximize profit and minimize loss to the investors. Financial derivatives are two way instruments which can be used to hedge risk for one party of a contract and simultaneously, it can provide opportunities of high returns to the other party involved in the financial contract. Derivatives are used as tools to reduce prominent risks in a variety of financial products such as uncertainty in the value of stock, bond, commodity, index prices and changes in foreign exchange rates, etc.. The most common forms of financial derivatives are various types of options such as European option, American option, and Asian option.

In this paper, we will focus on vanilla (European and American put options) and exotic options (Digital call and Butterfly options). Exotic options are not like other standardized options and these options are widely used in financial contracts. Exotic options embody special features which are designed in such a manner to meet the specific needs of investors.

Standard vanilla option is the European option that can be exercised at the expiry date only. In contrast, the American option can be exercised on or before the expiry date. Due to this flexibility, American options are more popular among the investors. However, this advantage is charged by the extra computational cost of finding not only the option value at each time step but also its exercising option as well. In mathematical terms, these factual dynamics turn the American option model into a moving boundary problem. In order to deal with the moving boundary model of the American put option, a penalty source term approach is used. In this procedure [22], a small continuous penalty term is added to the Black–Scholes equation [6] in order to get rid of the free boundary. Consequently, the model is transformed into a fixed boundary value problem. Various numerical methods such as [5,15,40,25,42–44] are utilized for solving American options.

Due to the importance of computational finance and option pricing, a lot of research has been focused on modelling and simulation of the financial derivatives. Black–Scholes model [6], a pioneering contribution, is a convection diffusion type of PDE model (based on the assumption that the stock price follows a Brownian motion, using the risk neutral probability) has been in use in many forms to model options pricing phenomenon. Closed-form

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solution for some options exist, however, most of the realistic models with complicated dynamics are solved numerically. Consequently, efficient and accurate numerical methods are required for the correct evaluation of such type of option pricing PDE models.

Existing literature contains a variety of numerical methods like finite difference methods [2,13,38,37] and finite element methods [4,40,44]. These methods are successfully applied in the field of computational finance and the simulation results are usually reasonably accurate. The recent work [2], which uses an efficient extrapolation technique for one dimensional European and Digital options, has attained superior accuracy. However, due to ease of implementation in higher dimensions on both scattered and uniform nodes, spectral accuracy and simplicity in coding, RBF based numerical methods are the potential candidates and are used as an alternative numerical tool. Meshless methods have been investigated by several researchers [3,5,9–11,15,23,24] for the correct evaluation of PDE models in computational finance. In [25], the authors have proposed radial basis point interpolation (RBPI) method to solve the Black–Scholes model for one asset European and American options. In that paper, the authors have introduced several numerical methods, namely: an exponential change of variables, a mesh refinement algorithm, and an implicit Euler Richardson extrapolated scheme.

In the last two decades, the meshless methods have been extensively used for the numerical approximations of different types of PDEs. The recent surge in the use of meshless methods is due to meshless character and ease of handling of multi-dimensional PDEs by these methods. Meshless methods are easily extendable up to three dimensional PDEs. However, in the case of fourth-dimensional spatio-temporal PDEs and beyond, the computational complexity of the meshless methods increase like other traditional numerical methods, due to computational efforts involved in the proper selection of stencil, choice of efficient linear solver and requirements on memory storage, which is directly related to the number of unknowns and the number of equations of the consequent system of algebraic equations. The use of RBF based algorithms to solve some attractive and interesting models can be found in [1,10,11,21,26,27,30–32,34,35].

Like other numerical methods, global meshless methods based on shape parameter dependent RBFs also bear some deficiencies like dense ill-conditioned matrices and non-availability of a well-defined procedure for selecting the optimum value of the shape parameter. To circumvent the problems of ill-conditioning and minimizing shape parameter sensitivity in the absence of optimum value, meshless methods based on local interpolation are thought to be more appropriate to get an accurate and a stable solution of the PDE models. This is due to the reason that the shape parameter dependent global meshless methods are highly sensitive to variation of shape parameter c and the choice of a good or a bad value of the shape parameter c deeply affects accuracy and stability of the method.

In this paper, the problem has been addressed by switching from global meshless methods to local meshless methods. Local meshless methods are far less sensitive to the variation of the shape parameter than the global counter parts. In the last few years, hidden potentials and capabilities of different types of local meshless methods have been explored in a variety of applications. The local meshless methods have proven themselves to be an effective alternative tool and have been used accurately in solving complex PDEs (see [33,26,27,29,39,17,14] and the references therein).

Second contribution of the current work is that a range of time integrators mentioned in the abstract (with and without different splitting procedures) are coupled with the local meshless method. Both adaptive and non-adaptive time integrators are taken into account and their performance comparison analysis in the context of solving stiff systems is performed. Computational stability analysis of local and global meshless methods is performed as well. To the best of our knowledge, the collective performance analysis of different time integrators and their coupling with the meshless methods has not been performed on a single platform so far.

The organization of the rest of the paper is as follows: in Section 2, we describe the models briefly. In Section 3, we highlight the proposed method. In Section 4, we perform time discretization and in Section 5, computational stability analysis of local and global methods are discussed. in Section 6, the numerical methods are applied to different test problems and the results are compared with published work. In Section 7, some conclusions are drawn.

2. Black–Scholes option models

In this section, we give a short description of the option price models. These include: a single asset American put option, a single asset European put option, a single asset butterfly spread call option, two asset butterfly spread call option, a single asset digital call option and a two asset digital call option.

2.1. A single asset option models

The Black–Scholes PDE model can be written as

$$\frac{\partial V}{\partial \tau}+\mathcal{L}V(S,\tau)=0 \quad \tau<T,\ \ S\in\mathbf{R}, \tag{1}$$

where

$$\mathcal{L}V(S,\tau)=\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2}+(r-D)S\frac{\partial V}{\partial S}-rV. \tag{2}$$

The variable S is a space variable showing the price of the underlying asset, r is the risk free interest rate, D is the dividend paid by the asset, σ is the volatility of underlying asset and $V(S,\tau)$ is the value of the option before the expiry time T.

The early exercise constraint leads to the following boundary and terminal conditions [25,41]:

$$\begin{aligned}
&V(S,T)=\max(E-S,0), \quad S\geq 0,\\
&\frac{\partial V}{\partial S}(\overline{S},\tau)=-1,\\
&V(\overline{S}(\tau),\tau)=E-\overline{S}(\tau),\\
&\lim_{S\to\infty}V(S,\tau)=0,\\
&\overline{S}(T)=E,\\
&V(S,\tau)=E-S, \quad 0\leq S<\overline{S}(\tau),
\end{aligned} \tag{3}$$

where $\overline{S}(\tau)$ represents the free boundary.

Since early exercise is permitted, the value V of the option must satisfy [38]

$$V(S,\tau)\geq\max(E-S,0), \quad S\geq 0,\ \ 0\leq\tau\leq T. \tag{4}$$

Using the penalty term approach [45,22], Eq. (1) is converted into a nonlinear partial differential equation on a fixed domain in the following form:

$$\frac{\partial V}{\partial \tau}+\mathcal{L}_\mu V(S,\tau)=0, \quad 0\leq S\leq S_\infty,\ \ 0\leq\tau\leq T \tag{5}$$

where

$$\mathcal{L}_\mu V(S,\tau)=\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2}+(r-D)S\frac{\partial V}{\partial S}-rV+\frac{\mu C}{V+\mu-q(S)}. \tag{6}$$

The above PDE is coupled with the following final and boundary conditions on a fixed domain:

$$\begin{aligned}
&V(S,T)=\max(E-S,0),\\
&V(0,\tau)=E,
\end{aligned}$$