Engineering Analysis with Boundary Elements 58 (2015) 7–17

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

An element-free IMLS-Ritz framework for buckling analysis of FG–CNT reinforced composite thick plates resting on Winkler foundations

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ARTICLE INFO

Article history:
Received 11 September 2014
Received in revised form
2 February 2015
Accepted 4 March 2015
Available online 24 March 2015

Keywords:
Buckling
Elastic foundation
Functionally graded carbon nanotube-reinforced composites
First-order shear deformation theory
Element-free IMLS-Ritz method

ABSTRACT

An element-free based improved moving least squares-Ritz (IMLS-Ritz) method is proposed to study the buckling behavior of functionally graded nanocomposite plates reinforced by single-walled carbon nanotubes (SWCNTs) resting on Winkler foundations. The first-order shear deformation theory (FSDT) is employed to account for the effect of shear deformation of plates. The IMLS is used for construction of the two-dimensional displacement field. We derive the energy functional for moderately thick plates. By minimizing the energy functional via the Ritz method, solutions for the critical buckling load of the functionally graded carbon nanotube (FG–CNT) reinforced composite plates on elastic matrix are obtained. Numerical experiments are carried out to examine the effect of the Winkler modulus parameter on the critical buckling loads. The influences of boundary condition, plate thickness-to-width ratio, plate aspect ratio on the critical buckling loads are also investigated. It is found that FG–CNT reinforced composite plates with top and bottom surfaces of CNT-rich have the highest critical buckling loads.

1. Introduction

The exceptional mechanical properties of carbon nanotubes (CNTs), i.e. extraordinary high elastic modulus, stiffness-to-weight and strength-to-weight ratio, have made them an excellent candidate for the reinforcement of polymer composites [1,2]. Motivated by their wide application prospect, CNT-reinforced composites have attracted much attention of researchers. The precise mechanical properties of CNT-reinforced composites have been extensively investigated experimentally and theoretically. These studies include but not limited to the analysis of elastic moduli, thermo-mechanical properties, bending, vibration and buckling behaviors of CNT-reinforced composite structures, like CNT-reinforced beams, plates or shells [3–7].

Inspired by the concept of functionally graded materials (FGMs), researchers adopted the functionally graded (FG) pattern of reinforcement for FG–CNT reinforced composite plates in their study. Shen [8] first studied the nonlinear bending of FG–CNT reinforced composite plates in thermal environment. He found that the load bending moment curves of the plates can be considerably improved through the use of a functionally graded distribution of CNTs in the matrix. Wattanasakulpong et al. [9] provided analytical solutions for bending, buckling and vibration responses of CNT-reinforced composite beams resting on an elastic foundation. Yas et al. [10] investigated free vibration and buckling behaviors of nanocomposite Timoshenko beams reinforced by single-walled carbon nanotubes (SWCNTs) on an elastic foundation. Shen and Zhang [11,12] reported thermal buckling and postbuckling behaviors of FG–CNT reinforced composite plates and shells subjected to in-plane temperature variation. Shen et al. [13] also considered the postbuckling of axially compressed nanotube-reinforced composite cylindrical panels resting on elastic foundations in thermal environments.

Although some researches on FG–CNT reinforced composite plates have been reported, FG–CNT reinforced composite is a newly proposed composite material and more research efforts need to be made for the better application of this new material. In term of mechanical analysis, a critical challenge is to develop efficient and accurate numerical methods for approximate solution of FG–CNT reinforced composite plates. Apparently, buckling analysis of the problem can be furnished by using any discretization techniques such as the finite element (FE) method [14]. Besides, there are other discretization techniques such as the NURBS-based FE method [15–17] and the isogeometric approach [18,19] have been proposed for analysis of composite plates as well as others [20,21]. Different from the FE method, the element-free

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http://dx.doi.org/10.1016/j.enganabound.2015.03.004
0955-7997/

methods [22–29] eliminate meshes by constructing the approximation entirely in terms of nodes. The FSDT element-free method has been used to perform the bending analysis of folded laminated plate structures [30]. The local Kriging meshless method has been employed to carry out free vibration analysis of moderately thick functionally graded plates [31]. The kp-Ritz method has been utilized to study the dynamic stability, large deflection, buckling and postbuckling behaviors of FG–CNT reinforced composite plates and panels [7,22,32–34].

The aim of this paper is to establish a simple and suitable computational model to investigate the buckling behavior of FG–CNT reinforced composite plates resting on Winkler foundations. The element-free based IMLS-Ritz method is used to compute the critical buckling loads of the FG–CNT reinforced composite plates. The effective material properties of the FG–CNT reinforced composite plates are estimated by the extended rule of mixture. The influence of Winkler's modulus parameters on the buckling behavior of FG–CNT reinforced composite plates is examined. Also the effects of CNT volume fraction, CNT distribution, plate thickness-to-width ratio, plate aspect ratio on the FG–CNT reinforced composite plates are examined under different in-plane loads.

2. Material properties of CNT-reinforced composite plates

CNT-reinforced composite plates, having three types of distributions of CNTs, with length a, width b and thickness h are considered. Material properties of the FG–CNT reinforced composites are assumed to be graded through thickness direction according to a linear distribution of the volume fraction of carbon nanotubes. Distributions of CNTs along the thickness direction of FG–CNT reinforced composite plates, as shown in Fig. 1, are assumed to be

$$V_{\mathrm{CNT}}(z)=\begin{cases} V^*_{\mathrm{CNT}} & (\mathrm{UD}) \\ 2\left(1-\frac{2|z|}{h}\right)V^*_{\mathrm{CNT}} & (\mathrm{FG}-O) \\ 2\left(\frac{2|z|}{h}\right)V^*_{\mathrm{CNT}} & (\mathrm{FG}-X) \end{cases},$$

where UD represents a uniform distribution and the other two types of functionally graded distributions of CNTs are denoted by FG-O and FG-X, respectively. The CNT volume fraction of UD FG–CNT reinforced composite plate and the other two types of FG–CNT reinforced composite plates are assumed to be $V_{\mathrm{CNT}}=V^*_{\mathrm{CNT}}$, that means all these three types of FG–CNT reinforced composite plates having the same mass volume of CNTs.

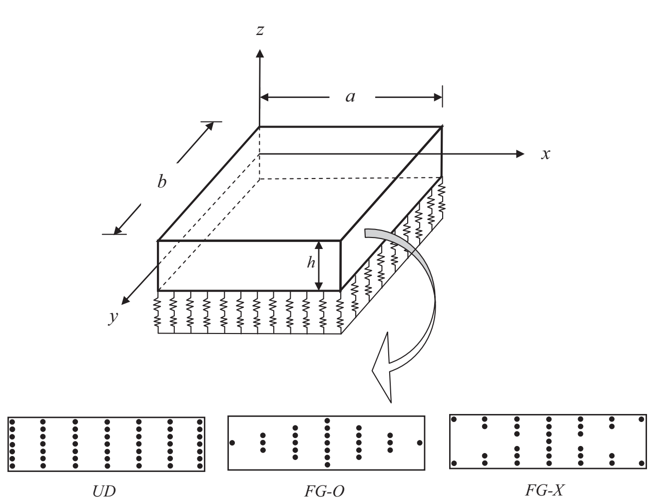

Fig. 1. Geometry and configurations of FG–CNT reinforced composite plates on Winkler foundation.

An embedded carbon nanotube in a polymer matrix is considered, therefore, there is no abrupt interface between the CNT and the polymer matrix. It is assumed the FG–CNT reinforced composite plates are made of a mixture of SWCNTs and an isotropic matrix. The rule of mixture is employed to estimate the effective material properties of FG–CNT reinforced composite plates. The poly{(*m*-phenylenevinylene)-*co*-[(2,5-dioctoxy-*p*-phenylene) vinylene]}, referred to as PmPV, is selected for the matrix [12]. The (10, 10) armchair SWCNTs are used as the reinforcements. The detailed material properties of SWCNTs used for the present analysis of the FG–CNT reinforced composite plates are selected from the simulation results reported by Shen and Zhang [12] and they are tabulated in Table 1.

3. Theoretical formulations

3.1. Equations of motion

The physical system considered is a FG–CNT reinforced composite plate of length a, width b, thickness h, with an arbitrary combination of boundary conditions along the four edges, as shown in Fig. 2. The FG–CNT reinforced composite plate is resting on an elastic foundation whose supporting action is described by the Winkler model [35] in this study, i.e.

$$q=K_w w, \tag{1}$$

in which q is the foundation reaction per unit area, K_w is the Winkler modulus of the elastic medium and w is the transverse deflection of the plate.

The stain energy contributed by the Winkler elastic medium is given by

$$U_m=\frac{1}{2}\int_{\Omega} K_w \cdot w^2 d\Omega. \tag{2}$$

Table 1
Comparison of Young's moduli for PmPV/CNT composites reinforced by (10, 10) SWCNT under $T=300$ K.

V^*_{CNT}	MD [12]		Rule of mixture			
	E_{11} (GPa)	E_{22} (GPa)	E_{11} (GPa)	η_1	E_{22} (GPa)	η_2
0.11	94.8	2.2	94.57	0.149	2.2	0.934
0.14	120.2	2.3	120.09	0.150	2.3	0.941
0.17	145.6	3.5	145.08	0.149	3.5	1.381

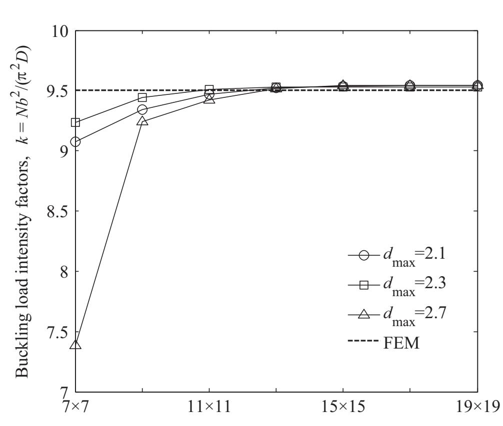

Fig. 2. Convergence of buckling load intensity factors, $k=Nb^2/(\pi^2 D)$, for simply supported FG–CNT reinforced composite plates with $a=b=10$ in., and $t=0.2$ in., on an elastic foundation.