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The topology optimization design for cracked structures

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ABSTRACT

In this paper, the element free Galerkin method (EFG) is proposed for topology optimization of cracked structures using the bi-directional evolutionary structural optimization method (BESO). The mathematical formulation of the topology optimization is developed considering the nodal strain energy as the design variable and the minimization of compliance as the objective function. The element free Galerkin method is enriched by the crack-tip enrichment functions to increase the approximation accuracy near the crack-tip. The Lagrange multiplier method is employed to enforce the essential boundary conditions. Several numerical examples are presented to show the effectiveness of the proposed method. Many issues related to topology optimization of cracked structures such as the effects of crack size and location on the optimal topology are addressed in the examples. The common numerical instabilities do not exist in the results.

1. Introduction

The structural optimization techniques have made remarkable progress in recent years. The objective of the structural optimization is to minimize the structural weight while maintaining the overall stiffness and satisfying the design constraints in a specific design domain [1]. The optimization methods can be employed in order to optimize the structural size, shape and topology. In size and shape optimization, only the cross-sectional properties and the design domain boundaries are optimized respectively, while the topology optimization in addition to variation of the design domain boundaries specifies number, size and location of cavities in continuum structures. The topology optimization remarkably optimizes the structural performance and has great practical application in the design of structures such as bridges and buildings. Several topology optimization methods have been proposed in the past decades to meet the structural requirements. The homogenization method and its variant, the variable density approach are one of the most popular approaches in the field of structural topology optimization [2]. The variable density approach has attracted many researchers due to its clear concept and simplicity implementation. Other topology optimization approaches have also been constructed and developed. The level set method was created to change the structure outline for finding an optimal shape design [3]. The evolutionary structural optimization method (ESO) was proposed based on gradual elimination of inefficient materials [4]. In the ESO method, because the inefficient materials are completely removed from the structure, there is no information about the effects of these materials on the objective function in the later stages of optimization. The bi-directional ESO method (BESO) is a further development of the ESO method that is developed based on the idea of adding the efficient materials to the structure as well as removing the inefficient ones [5–7]. The BESO method has many advantages over the ESO method in terms of computational efficiency, robustness of the method and manufacturability of the final topology [8]. The significance of the BESO method is its simplicity and applicability for optimization of various types of structures [9–12].

It should be noted that almost in all the previous research work [1–12] the numerical method is the finite element method (FEM). The FEM encounters some difficulties when dealing with problems involving large deformation, moving boundaries and crack propagation due to the need for frequent remeshing. Especially, crack propagation is a prime example in which the use of FEM requires a large number of remeshings of the finite element model to represent arbitrary and complex paths. To overcome these difficulties, a group of meshfree methods such as smooth particle hydrodynamics (SPH), element free Galerkin (EFG), reproducing kernel particle (RKPM) and meshless local Petrov–Galerkin method have been found. Due to their meshfree nature, these methods do not require maintaining the integrity and desired shape of the elements. Therefore, the crack propagation can be effectively simulated with meshless methods.

Especially, the element free Galerkin method is very versatile for modeling of propagation of cracks, due to the absence of any predefined element connectivity and no need to frequent remeshings. The EFG utilizes moving least-squares interpolants which requires only nodes, unencumbered by elements and elemental connectivity, to construct the shape functions. It has been found that the EFG has a good convergence rate and high order continuity of the field variables

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that generally exhibits good numerical stability and accuracy for computational solid mechanics problems [13,15,16,29]. This method has mainly been applied in the area of crack propagation, where nodes were continuously moved or added to follow the crack tip [14,25].

To date, a few researchers have employed meshless methods to topology optimization of continuum structures. The ESO method integrated with the meshless Galerkin method was applied to carry out the topology optimization of continuum structures [16]. The meshless Galerkin method was combined with a meaningful density approximant to create an efficient topology optimization approach [17]. The variable density approach based on the RKPM method was developed for topology optimization of geometrically nonlinear structures [18]. The implicit topology description function was integrated into the RKPM method to implement the topology optimization of continua [19]. The structural shape and topology optimization problem was solved by using the level set method incorporated with the Galerkin global weak forms [20]. The EFG method combined with the Pareto-optimality theory was applied to carry out the topology optimization of continuum structure [21]. The BESO method based on the EFG method was developed for topology optimization of continuum structures [22]. By far, we have not found any publication that introduces meshfree methods into the topology optimization to design the cracked structures.

In this paper, the element free Galerkin method (EFG) is proposed to solve the topology optimization problem of cracked structures using the bi-directional evolutionary structural optimization method (BESO). The mathematical formulation of the topology optimization is developed considering the nodal strain energy as the design variable and the minimization of compliance as the objective function. The EFG is enriched by the crack-tip enrichment functions to increase the approximation accuracy near the crack-tip. Several 2D examples that are widely used in topology optimization problems are illustrated to show the effectiveness of the proposed method. The examples show that the present method can effectively suppress the common numerical instabilities such as checkerboard patterns. Many issues related to topology optimization of cracked structures such as the effects of crack size and location on the optimal topology are addressed in the examples.

2. Review of the element free Galerkin method (EFG)

In this section a brief description of the EFG method is given, full details can be found in References [13,23,26]. In the EFG method, the approximation $u^h(\mathbf{x})$ for the displacement field $u(\mathbf{x})$ can be written as:

$$u_i(\mathbf{x}) = u_i^h(\mathbf{x}) = a_{ij}(\mathbf{x})p_j(\mathbf{x}) \tag{1}$$

where a_{ij} are unknown parameters. The parameters a_{ij} at any given point $\mathbf{x}$ are determined by minimizing the least-squares functional:

$$J_i(\mathbf{x}) = \sum_{I=1}^{n} w(\mathbf{x}-\mathbf{x}_I)[a_{ij}(\mathbf{x})p_j(\mathbf{x}_I) - u_{iI}]^2 \tag{2}$$

where $w(\mathbf{x}-\mathbf{x}_I)$ is the weight function associated with node $\mathbf{x}_I$, u_I are the nodal displacement parameters and n is the number of nodes whose support includes point $\mathbf{x}$. The minimization of $J(\mathbf{x})$ in Eq. (2) leads to linear relations between a_{ij} and u_{iI} so that Eq. (1) can be written in standard shape function as follows:

$$u_i^h(\mathbf{x}) = \sum_{I=1}^{n} \Phi_I(\mathbf{x})u_{iI} \tag{3}$$

It should be noted that the shape functions derived from the moving least-squares (MLS) approximation do not satisfy the Kronecker delta condition ($\Phi_I(\mathbf{x}_J) \neq \delta_{IJ}$). Therefore, extra procedures such as Lagrange multipliers are needed for the imposition of the essential boundary conditions.

2.1. Discrete equations and Lagrange multipliers

Consider a two-dimensional problem on the domain Ω bounded by Γ in elasticity mechanics. In this problem, $\mathbf{u}$ denotes the displacement vector on Ω, $\mathbf{b}$ is the body force vector, $\bar{\mathbf{u}}$ is the prescribed displacement on boundary Γ_u and $\bar{\mathbf{t}}$ is the prescribed traction on boundary Γ_t. In the EFG, the final discrete equation for this problem is [13]:

$$\begin{bmatrix} \mathbf{K} & \mathbf{K} \\ \mathbf{G}^T & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{q} \end{Bmatrix} \tag{4}$$

where

$$\mathbf{K}_{IJ} = \int_{\Omega} \mathbf{B}_I{}^T \mathbf{D} \mathbf{B}_J d\Omega; \quad \mathbf{G}_{IK} = -\int_{\Gamma_u} \Phi_I N_K d\Gamma; \quad \mathbf{f}_I = \int_{\Omega} \Phi_I \mathbf{b} d\Omega + \int_{\Gamma_t} \Phi_I \bar{\mathbf{t}} d\Gamma \tag{5}$$

and

$$\mathbf{q}_K = -\int_{\Gamma_u} N_K \bar{\mathbf{u}} d\Gamma; \quad \mathbf{B}_I = \begin{bmatrix} \Phi_{I,x} & 0 \\ 0 & \Phi_{I,y} \\ \Phi_{I,y} & \Phi_{I,x} \end{bmatrix}; \quad \mathbf{N}_K = \begin{bmatrix} N_K & 0 \\ 0 & N_K \end{bmatrix} \tag{6}$$

In the above equations, $\mathbf{B}_I$ is the standard strain matrix for node I, $\mathbf{N}_K$ is the Lagrange interpolation matrix and $\mathbf{D}$ is the stress-strain relationship matrix, respectively. The solution of Eq. (4) is the vector $\mathbf{u}$, which is a vector of nodal displacements. Subsequently, the nodal strain energy, α^{SE}, can be obtained using the strain-displacement and constitutive equations:

$$\alpha^{SE} = \frac{1}{2}\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\varepsilon} = \sum_{I=1}^{n} \frac{1}{2}\mathbf{u}_I{}^{\mathrm{T}} \mathbf{B}_I^T \mathbf{D} \mathbf{B}_I \mathbf{u}_I \tag{7}$$

2.2. Enriched EFG for modeling of crack

A crack is modeled in the EFG by defining a line segment internal to the domain. The main idea to capture the crack is to enrich the test and trial functions with additional unknowns so that the approximation is continuous in whole domain but discontinuous along the crack. Therefore, the test and trail functions are written in terms of a signed distance function f (see Fig. 1) [24-26]:

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in S} \Phi_I(\mathbf{x})\mathbf{u}_I + \sum_{I \in S^c} \Phi_I(\mathbf{x})H(f_I(x))\mathbf{a}_I + \sum_{I \in S^f} \Phi_I(\mathbf{x}) \sum_{\alpha=1}^{4} \mathbf{b}_K^{\alpha B_\alpha(\mathbf{x})} \tag{8}$$

where Φ_I are the MLS shape function and H and B_α are enrichment functions. The coefficient $\mathbf{a}$ and $\mathbf{b}$ are additional unknowns introduced for the crack in the variational formulation. The Heaviside function, H depends on the signed distance $f_I(\mathbf{x})$ and is defined as:

$$\begin{cases} H(f_I(\mathbf{x})) = 1 \text{if} f_I(\mathbf{x}) > 0 \\ H(f_I(\mathbf{x})) = -1 \text{if} f_I(\mathbf{x}) < 0 \end{cases} \tag{9}$$

with

$$f_I(\mathbf{x}) = \begin{cases} \text{sign}[\mathbf{n}.(\mathbf{x}_I - \mathbf{x})\min\|\mathbf{x}_I - \mathbf{x}\|, \text{for } \mathbf{x}_I \in S^c \\ \mathbf{n}.(\mathbf{x}_{\text{tip}} - \mathbf{x}_I), \text{for } \mathbf{x}_I \in S^f \end{cases} \tag{10}$$

where $\mathbf{n}$ is the crack normal and $\mathbf{x}_{\text{tip}}$ are the coordinates of the crack tip. In the linear elastic fracture mechanics, $\mathbf{B}$ is chosen to be continuous in the whole domain S^f, but discontinuous at the crack