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Stochastic spline fictitious boundary element method for analysis of thin plate bending problems with random fields

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ABSTRACT

Mathematical formulation and computational implementation of the stochastic spline fictitious boundary element method (SFBEM) are presented for stochastic analysis of thin plate bending problems with loadings and structural parameters modeled with random fields. Two sets of governing differential equations with respect to the mean and deviation of deflection are derived by including the first order terms of deviations. These equations are in similar forms to those of deterministic thin plate bending problems, and can be solved using deterministic fundamental solutions. The calculation is conducted with SFBEM, a modified indirect boundary element method (IBEM), resulting in the means and covariances of responses. The proposed method is validated by comparing the solutions obtained with Monte Carlo simulation for a number of example problems and a good agreement of results is observed.

1. Introduction

The structural problems with random parameters appear frequently in engineering practices due to the stochastic nature of the loadings as well as the material properties and geometric parameters. It is obvious that a deterministic analysis cannot capture the random nature of the structural response, since only expectation values rather than statistical information of the parameters are used in the computational model. Numerical computation schemes in the frame of finite element method (FEM) have been developed to deal with the problems modeled with statistical nature of loads, material properties and geometric parameters [1,2]. Due to some inherent unique advantages [3], the boundary element method (BEM) can be used as an alternative numerical approach to FEM for stochastic problems and has been used in various areas [4].

Similar to the perturbation stochastic FEM, the stochastic BEM is most commonly performed in conjunction with the perturbation method. The methodology was first applied in plane elasticity problems with random geometric configuration by Nakagiri et al. [5] using the second-order perturbation method in combination with the conventional BEM. Later, Kaljević and Saigal [6] adopted the same method for elastostatic problems by modeling configuration as random variable and material property as random field respectively. Kamiński has conducted various studies by using the perturbation techniques in stochastic BEM [7–9]. The implementation of perturbation BEM and its modified methods have also been available for stochastic groundwater flow problems [10], stochastic seawater intrusion problems [11], stochastic potential problems [12,13], stochastic heat transfer problems [14], stochastic wave propagation problems [15], and dynamical problems [16]. Recently, a stochastic BEM based on the first order approximation has been proposed by the authors for stochastic analysis of elastostatic problems with the material properties modeled with random fields [17].

In this paper, SFBEM is extended to stochastic analysis of thin plate bending problems with random fields. SFBEM is a modified approach to the conventional IBEM. In SFBEM, nonsingular integral equations are derived using the fictitious boundary techniques, and spline functions with excellent performance are adopted as the trial functions to the unknown fictitious loads. Then the boundary-segment-least-square technique is employed in SFBEM for eliminating the boundary residues, which leads to the numerical solution to the integral equations. Because of these modifications, SFBEM is of high accuracy and efficiency in general. The method was first applied to the solution of static plane elasticity problems [18], and so far it has been extended to multi-domain plane problems [19], orthotropic plane problems [20], stochastic elastostatic problems [17,21], and stochastic fracture problems [22].

First order approximation technique is adopted in the proposed method of this paper, which is devoted to the calculation of random solutions for thin plate bending problems with varied loads, material properties and plate thickness modeled by random fields. Different from the general perturbation BEM, the input parameters

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and output responses are expressed with the mean and deviation values. They are then substituted into the stochastic governing differential equations and relationship formulations between internal forces and deflection, and the second and higher order terms of the deviation values are neglected, leading to two sets of equations described with the mean and deviation value of deflection respectively. As these equations have similar forms as those in the deterministic problems, they can be solved by BEM with deterministic fundamental solutions. In particular, the SFBEM is applied in the present method for calculations of the mean and deviation equations to achieve statistical values of responses with high accuracy and efficiency. The feasibility and effectiveness of the present approach are validated using several numerical examples. A good agreement of results can be observed by comparison with the Monte Carlo simulation.

2. Stochastic governing differential equation

Consider a thin plate with varied bending rigidity $D(x,y)$, subjected to a transverse load $q(x,y)$. The governing differential equations for this plate bending problem can be expressed by deflection as

$$D\nabla^4 w+2\frac{\partial D}{\partial x}\frac{\partial \nabla^2 w}{\partial x}+2\frac{\partial D}{\partial y}\frac{\partial \nabla^2 w}{\partial y}+(\nabla^2 D)(\nabla^2 w) -(1-\mu)\left(\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial x^2}\right)=q \tag{1}$$

where μ is Poisson's ratio; (x,y) is the Cartesian coordinate of a point in the plate domain considered; and w is the deflection of the plate.

Assuming $q(x,y)$ is a general random field and $D(x,y)$ is a homogeneous one, they can be expressed as

$$\left.\begin{aligned} q(x,y)&=q_E(x,y)+\delta q(x,y)\\ D(x,y)&=D_E+\delta D(x,y)\end{aligned}\right\} \tag{2}$$

where $q_E(x,y)$ and D_E are the mean values of $q(x,y)$ and $D(x,y)$, respectively, with D_E being a constant; and $\delta q(x,y)$ and $\delta D(x,y)$ represent the deviations of $q(x,y)$ and $D(x,y)$, respectively, with their mean values being zero. Accordingly, the deflection and internal forces are also random fields and can be written as

$$\left.\begin{aligned} w(x,y)&=w_E(x,y)+\delta w(x,y)\\ Q_x(x,y)&=Q_{xE}(x,y)+\delta Q_x(x,y)\\ Q_y(x,y)&=Q_{yE}(x,y)+\delta Q_y(x,y)\\ M_x(x,y)&=M_{xE}(x,y)+\delta M_x(x,y)\\ M_y(x,y)&=M_{yE}(x,y)+\delta M_y(x,y)\\ M_{xy}(x,y)&=M_{xyE}(x,y)+\delta M_{xy}(x,y)\end{aligned}\right\} \tag{3}$$

where Q_x and Q_y are shear forces; M_x and M_y are bending moments; M_{xy} is torsional moment; $(\bullet)_E$ represents the mean value, and $\delta(\bullet)$ represents the deviation with the mean value equaling to zero.

Substituting Eqs. (2) and (3) into Eq. (1), and neglecting the second order terms of the deviations and their derivatives, one can obtain

$$D_E\nabla^4 w_E+\delta D\nabla^4 w_E+D_E\nabla^4\delta w+2\frac{\partial\delta D}{\partial x}\frac{\partial\nabla^2 w_E}{\partial x}+2\frac{\partial\delta D}{\partial y}\frac{\partial\nabla^2 w_E}{\partial y}+(\nabla^2\delta D)(\nabla^2 w_E) -(1-\mu)\left(\frac{\partial^2\delta D}{\partial x^2}\frac{\partial^2 w_E}{\partial y^2}-2\frac{\partial^2\delta D}{\partial x\partial y}\frac{\partial^2 w_E}{\partial x\partial y}+\frac{\partial^2\delta D}{\partial y^2}\frac{\partial^2 w_E}{\partial x^2}\right)=q_E+\delta q \tag{4}$$

Taking the expectances of terms on both sides of Eq. (4), one has

$$D_E\nabla^4 w_E=q_E \tag{5}$$

Substitution of Eq. (5) into Eq. (4) yields

$$D_E\nabla^4\delta w=\delta f \tag{6}$$

where $\delta f(x,y)$ is the equivalent transverse load and can be expressed as

$$\delta f=\delta q-\delta D\nabla^4 w_E-2\frac{\partial\delta D}{\partial x}\frac{\partial\nabla^2 w_E}{\partial x}-2\frac{\partial\delta D}{\partial y}\frac{\partial\nabla^2 w_E}{\partial y}-\frac{\partial^2\delta D}{\partial x^2}\left(\frac{\partial^2 w_E}{\partial x^2}+\mu\frac{\partial^2 w_E}{\partial y^2}\right) -\frac{\partial^2\delta D}{\partial y^2}\left(\frac{\partial^2 w_E}{\partial y^2}+\mu\frac{\partial^2 w_E}{\partial x^2}\right)-2(1-\mu)\frac{\partial^2\delta D}{\partial x\partial y}\frac{\partial^2 w_E}{\partial x\partial y} \tag{7}$$

Note that $D(x,y)$ can be expressed by the elasticity modulus $E(x,y)$ and the plate thickness $h(x,y)$ as

$$D(x,y)=\frac{E(x,y)[h(x,y)]^3}{12(1-\mu^2)} \tag{8}$$

Assuming $E(x,y)$ and $h(x,y)$ are homogeneous random fields, they can be expressed as

$$\left.\begin{aligned} E(x,y)&=E_E+\delta E(x,y)\\ h(x,y)&=h_E+\delta h(x,y)\end{aligned}\right\} \tag{9}$$

where E_E and h_E are the mean values of $E(x,y)$ and $h(x,y)$, respectively, with E_E and h_E being constants; and $\delta E(x,y)$ and $\delta h(x,y)$ represent the deviations of $E(x,y)$ and $h(x,y)$, respectively, with their mean values being zero. Substituting Eqs. (2) and (9) into Eq. (8), and neglecting the second and higher order terms of the deviations, one has

$$D_E+\delta D=\frac{E_E h_E^3+h_E^3\delta E+3E_E h_E^2\delta h}{12(1-\mu^2)} \tag{10}$$

Taking the expectances of terms on both sides of Eq. (10), one obtains

$$D_E=\frac{E_E h_E^3}{12(1-\mu^2)} \tag{11}$$

Substitution of Eq. (11) into Eq. (10) yields

$$\delta D=\frac{h_E^3\delta E+3E_E h_E^2\delta h}{12(1-\mu^2)} \tag{12}$$

It can be seen from Eqs. (11) and (12) that D_E and δD can now be expressed using the means and deviations of the elasticity modulus and the plate thickness, respectively. Accordingly, Eqs. (5)–(7) can also be expressed by the means and deviations of the elasticity modulus and the plate thickness using the above equations. Substituting Eq. (11) into Eq. (5), one has

$$E_E {h_E}^3\nabla^4 w_E=12(1-\mu^2)q_E \tag{13}$$

Substituting Eqs. (11) and (12) into Eqs. (6) and (7), one can obtain

$$E_E {h_E}^3\nabla^4\delta w=12(1-\mu^2)\delta f \tag{14}$$

and

$$\delta f=\delta q-\frac{h_E^3}{12(1-\mu^2)}\left[\delta E\nabla^4 w_E+2\frac{\partial\delta E}{\partial x}\frac{\partial\nabla^2 w_E}{\partial x}+2\frac{\partial\delta E}{\partial y}\frac{\partial\nabla^2 w_E}{\partial y} +\frac{\partial^2\delta E}{\partial x^2}\left(\frac{\partial^2 w_E}{\partial x^2}+\mu\frac{\partial^2 w_E}{\partial y^2}\right)+\frac{\partial^2\delta E}{\partial y^2}\left(\frac{\partial^2 w_E}{\partial y^2}+\mu\frac{\partial^2 w_E}{\partial x^2}\right)+2(1-\mu)\frac{\partial^2\delta E}{\partial x\partial y}\frac{\partial^2 w_E}{\partial x\partial y}\right] -\frac{E_E h_E^2}{4(1-\mu^2)}\left[\delta h\nabla^4 w_E+2\frac{\partial\delta h}{\partial x}\frac{\partial\nabla^2 w_E}{\partial x}+2\frac{\partial\delta h}{\partial y}\frac{\partial\nabla^2 w_E}{\partial y}+\frac{\partial^2\delta h}{\partial x^2}\left(\frac{\partial^2 w_E}{\partial x^2}+\mu\frac{\partial^2 w_E}{\partial y^2}\right)\right.$$