Engineering Analysis with Boundary Elements 58 (2015) 119–128

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Recursive moving least squares

Hamid Mehrabi*, Behzad Voosoghi

Department of Geodesy, Faculty of Geodesy and Geomatics, K.N. Toosi University of Technology, 19697-64499 Tehran, Iran

ARTICLE INFO

Article history:
Received 27 November 2014
Received in revised form
12 February 2015
Accepted 5 April 2015
Available online 24 April 2015

Keywords:
Meshless moving least squares
Recursive least squares
Scattered data points
Statistical test
Strain invariants
Surface approximation/interpolation
Variance–covariance matrix

ABSTRACT

The meshless moving least squares (MLS) is expanded here based on recursive least squares (RLS) where the outcome is the newly developed recursive moving least squares (RMLS) approximation method. In RMLS method each nodal point has its own size of the support domain; accordingly, the number of field points on the influence domain varies from node to node. This method makes it possible to select the optimal size of the support domain by imposing any arbitrary measures such as precision or convergence of the unknown parameters on the support domain. Moreover, the possibility of applying the statistical test in removing any undesired outliers of function values is provided. Another feature of this newly developed method is providing the possibility of revealing the significant break-lines and faults diagnosis on the surface. In RMLS the radius of the support domain would become extended to a point where the optimal precision of unknown parameters is achieved or reach the discontinuous or high gradient interfaces. The numerical results indicate that this method improves the accuracy of approximated surface more than 50%, especially for rough surfaces or the contaminated particles by random or gross errors, with no significant increase in time.

Contents

1. Introduction

The scattered data approximation/interpolation issue is known and is applied in many branches of science in general, geo-spatial analysis in specific. This issue is addressed in any discipline where measurements are to be taken at irregularly spaced values of two or more independent variables, especially prevalent in earth sciences and computer graphics.

The conventional numerical methods need a priori definition of the connectivity of the nodes, i.e. they rely on a mesh for interpolation or approximation of the property coordinates characteristics of the regular or irregular particles. The Finite Element Method (FEM), Finite Volume Method (FVM) and Finite Difference Method (FDM) are the most well-known elements of these thoroughly developed mesh-based methods.

In the recent decades meshless methods are developed as the alternatives to the widely used mesh-based methods. These comparable new class of numerical methods are developed to approximate the surfaces and partial differential equations only based on a set of nodes with no need for an additional mesh. A growing interest is observed in the development of computing techniques alternative to the FEM, promising faster convergence, smoother solutions and simpler discretization techniques [1]. In the meshless methods, the element definition is no longer needed

* Corresponding author.
E-mail address: hmehrabi@mail.kntu.ac.ir (H. Mehrabi).

http://dx.doi.org/10.1016/j.enganabound.2015.04.001
0955-7997/

to discretize the problem domain, here only nodal points' definition and boundary conditions of the domain are of concern.

A number of different functions suitable for application in the meshless methods are proposed, e.g., the Moving Least Squares (MLS) interpolation scheme [2], the Natural Element Method (NEM) [3], to name of few. A good method for creating meshless shape functions should be sufficiently robust, stable, regularized and compactly supported. Shape and size of the support domain (Fig. 1) and accuracy of approximation have always been challenging issues. Accuracy of approximation for a nodal point depends on the number and distribution of the field points in the support domain. In the meshless methods, a suitable support domain should be chosen to ensure an efficient and accurate approximation.

Zhuang and Heaney [4] found that the importance of selecting the size of nodal supports, which causes severe difficulties in terms of error control and solution accuracy. They also stated that the magnifier coefficient must be chosen so that sufficient nodes are in support for the basis used throughout the domain, and the scale factor has an important influence on the error numerically.

The issue of selecting the size of support is examined by Tsai and Guan [5] in order to explore the effect of number of particles and size of the compact support as a simple case, without any proposal to solve the number of field points. Deng and Tang [6] proposed a method to determine the number of nearest points using the interpolation error estimation. Wang and Liu [7] illustrated numerical examples where the field points from 6 to 17 could reach sufficient accuracy whether the nodes are structured or unstructured.

In some articles the size of the support domain is held fixed ([8–10]) and in the others the number of field points is held fixed throughout the entire domain. Almost none of the references have considered the nodal arrangement and distribution of the scattered field points throughout the influence domain, in addition to the continuity of the field functions throughout the support domain.

Crack diagnosis, modeling and growth is the underlying issue in the current research in many disciplines, especially in civil and mechanical engineering. The cracks are described by a jump in the displacement field for particles the influence domain of which is cut by the crack [11]. Rabczuk and Belytschko [12] developed a robust and simple in implementation cracking particle model, where the discontinuities are introduced at the particle positions. In Rabczuk and Zi [11] the cohesive cracks are demonstrated through local partition of unity and the influence domain of the discontinuous particles are enriched with branch functions through applying the extended concept of XFEM proposed by Ventura and Xu [13]. The particle methods by considering different cracking criteria are implemented by Rabczuk and Belytschko [14] for treating the crack growth in 3D. The 3D fracture modeling through meshless element-free Galerkin method, is adopted by [15] for stress analysis, where the level sets are used accurately to describe and capture crack evolution.

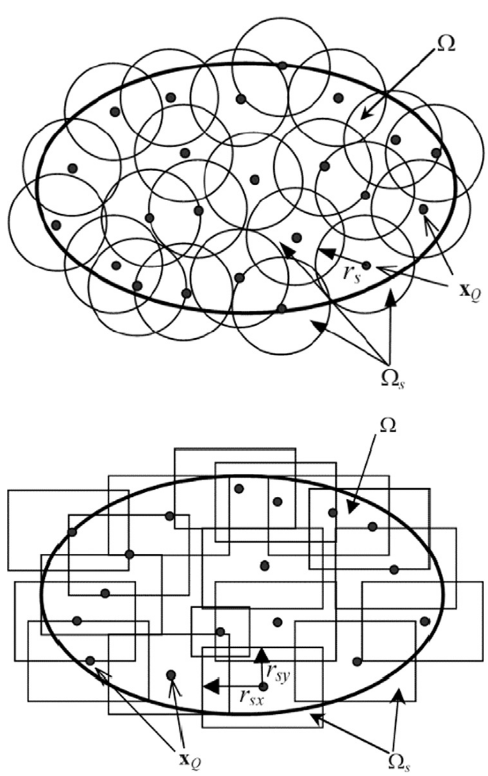

Fig. 1. Circular and rectangular support domains.

These methods are implemented to model cracks in finite element and meshfree methods. This newly developed RMLS could detect and reveal significant high gradient particle, cracks and break-lines on the approximated surfaces in a simple approach, that is, here no modeling is involved. One of the examples in the application of the RMLS is detecting the blind active faults in geology, through a statistical test of residuals vector as demonstrated in Section 3. In RMLS method, the size and shape of the support domain is not fixed throughout the entire domain of nodal points. It applies a dynamic procedure in selecting the field points. The dynamic procedure in selecting the size of the support domain based on some predefined criteria makes RMLS a robust and reliable method. This proposed method intrinsically considers both the continuity of the field functions and data arrangements throughout the support simultaneously.

The focus of this study is on investigating the meshless approximation method by applying the recursive least squares (RLS) estimation in order to overcome some drawbacks of the traditional meshless methods. A brief insight on the MLS, RLS and the advantages of their integration which developed RMLS is presented in Section 2. The rest of the article is organized as follows: Section 3 contains several examples for comparative studies and applications in practice in order to validate the capabilities of the RMLS by using some scattered data points in various situations. Concluding remarks appear in Section 4.

2. Recursive moving least squares

The newly developed RMLS in this paper is an integration of the well-known meshless approximation method, MLS and RLS. In this section the basic definitions and properties of the MLS and RLS and the advantages of this proposed method are presented.

2.1. Moving least squares (MLS)

The origin of meshfree methods could be traced back to a few decades (1960s, introduced by Shepard [16]), but it was not until after the early 1990s when substantial and significant advances were made in this field. This method is considered as the means of generating a smooth surface interpolating among various specified point values. The procedure was later extended for the same purpose by Lancaster and Salkauskas [2] and [17]. The MLS is a common procedure for scattered data approximation that applies the local polynomial fitting in the least squares sense. It takes advantage of simple calculation, high precision and smoothness [18]. The advantages of the MLS approximation is to obtain the shape function in meshless methods with higher order continuity and consistency by employing the basis functions with lower order continuity and choosing a suitable weight function [19].