Engineering Analysis with Boundary Elements 58 (2015) 151–165

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A simple accurate formula evaluating origin intensity factor in singular boundary method for two-dimensional potential problems with Dirichlet boundary

Xing Wei [a,b], Wen Chen [a,*], Linlin Sun [a], Bin Chen [a]

[a] State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, College of Mechanics and Materials, Hohai University, Nanjing 210098, China
[b] College of Civil Engineering and Architecture, East China Jiaotong University, Nanchang 330013, China

ARTICLE INFO

Article history:
Received 20 June 2014
Received in revised form
17 April 2015
Accepted 20 April 2015
Available online 16 May 2015

Keywords:
Singular boundary method
Origin intensity factor
Dirichlet boundary
Fundamental solution
Potential problems

ABSTRACT

In this work, a simple accurate formula is presented to evaluate the origin intensity factor of the singular boundary method (SBM) for two-dimensional Dirichlet potential problems. The SBM is considered as an improved version of the method of fundamental solutions and remedies the controversial auxiliary boundary outside the computational domain in the latter. The origin intensity factor is a central concept in the SBM to overcome the source singularity of the fundamental solution while placing source points on the physical boundary. In literature, the origin intensity factor for the Dirichlet boundary condition is numerically obtained which may cause numerical instability in large-scale simulations. This work proposes a simple formula to calculate the origin intensity factor for two-dimensional Dirichlet potential problems. Numerical experiments show that it is feasible and perform robustly for problems under various irregular domains.

1. Introduction

The method of fundamental solutions (MFS) [1,2] is a typical meshless boundary collocation method. The method is also known under several other names, such as the F-Trefftz method, a superposition method, a charge simulation method and a method of auxiliary source, etc. Like the boundary element method (BEM) [3,4], the MFS also employs the fundamental solution to reduce computational problem dimensionality by one and is readily applicable to boundary value problems (BVPs) in which the fundamental solution of the governing differential operator is accessible. The MFS avoids the numerical integration and mesh generation and is very attractive to solve high-dimensional problems [2,5–8]. However, in order to avoid the source singularity of fundamental solutions, the source points are positioned on an auxiliary boundary outside the computational domain of interest. The location of the auxiliary boundary is a perplexing and controversial issue associated with the traditional MFS, especially in practical problems with complex-shaped or multi-connected domains. Even though a lot of researchers in recent decades focus their study on this issue [2,9,10], an optimal placement in general is not achieved. Until now, the determination of the auxiliary boundary is largely based on practitioner's experience and trial-error approaches.

In recent years, considerable efforts are devoted to eliminating the troublesome auxiliary boundary in the MFS. Some papers [11,12] employ an alternative nonsingular kernel function to circumvent this problem, but the derivation of nonsingular functions for different problems [13] posses another challenge. On the other hand, some methods are proposed to remedy the source singularity with desingularization techniques, such as the regularized meshless method (RMM) [14,15], the modified method of fundamental solution (MMFS) [16], the boundary distributed source method (BDS) [17] and the singular boundary method (SBM) [18,19].

This study is concerned with the singular boundary method, in which the origin intensity factor (OIF) plays a central role to eliminate the fictitious boundary in the MFS. The existence of the OIF is firstly verified through diverse numerical experiments. In the literature, the inverse interpolation technique (IIT) [20,21] is a standard approach to evaluate the OIF. In the case of the Neumann boundary condition, a more accurate and efficient technique is proposed to evaluate the OIF via a subtracting and adding-back technique [22]. However, in case of the Dirichlet boundary condition, the IIT remains the method of choice. The IIT amounts to solving the matrix system twice and is thus computationally more expensive, in particular, in conjunction with fast algorithms

* Corresponding author. Tel.: +86 25 83786873; fax: +86 25 83736860.
E-mail address: chenwen@hhu.edu.cn (W. Chen).

http://dx.doi.org/10.1016/j.enganabound.2015.04.010
0955-7997/

[23,24] for dense matrix equations such as the fast multipole method for larger scale problems. In some cases, the IIT may also cause numerical instability.

This paper proposes a simple formula of the OIF for two-dimensional Dirichlet potential problems. Some numerical experiments are studied to show its robustness and feasibility to complex-shaped geometries. The present technique is numerically more stable than the IIT for Dirichlet potential problems and improves the efficiency of the SBM. The numerical results also demonstrate that the present SBM outperforms the modified method of fundamental solutions (MMFS) and the boundary distributed sources method (BDS) on accuracy and convergence.

The rest of the paper is constructed as following: in Section 2, the SBM for potential problems is formulated. In Section 3, a simple strategy is proposed for the determination of the OIF for Dirichlet boundary potential problems. Section 4 examines several benchmark numerical examples to illustrate the validity of the present OIF. And finally, some conclusions are summarized in Section 5.

2. SBM formulation

Let us consider a two-dimensional potential problem governed by Laplace equation

$$\nabla^2 u(\boldsymbol{x}) = 0, \quad \boldsymbol{x} = (x_1, x_2) \in \Omega \tag{1}$$

with boundary conditions

$$u(\boldsymbol{x}) = h_D(\boldsymbol{x}), \quad \boldsymbol{x} \subset \Gamma_D,$$
$$q(\boldsymbol{x}) = \frac{\partial u(\boldsymbol{x})}{\partial \mathbf{n}} = h_N(\boldsymbol{x}), \quad \boldsymbol{x} \subset \Gamma_N, \tag{2}$$

where u is the potential, q is the flux, $\boldsymbol{x}$ means the spatial coordinates, and $\boldsymbol{n}$ is the outward normal vector, $h_D(\boldsymbol{x})$ and $h_N(\boldsymbol{x})$ stand for measured data specified on the boundary, Ω is a computational domain as shown in Fig. 1 and $\partial\Omega = \Gamma_D \cup \Gamma_N$ denotes the whole physical boundary.

In the MFS, the solution of the problem (1)–(2) is expanded as a linear combination of the fundamental solution in terms of the source points as follows:

$$u(\boldsymbol{x}_i) = \sum_{j=1}^{N} \alpha_j G(\boldsymbol{x}_i, \boldsymbol{s}_j), \tag{3}$$

$$q(\boldsymbol{x}_i) = \sum_{j=1}^{N} \alpha_j \frac{\partial G(\boldsymbol{x}_i, \boldsymbol{s}_j)}{\partial \mathbf{n}}, \tag{4}$$

where $G(\boldsymbol{x}, \boldsymbol{s})$ is the fundamental solution of the governing equation, $G(\boldsymbol{x}, \boldsymbol{s}) = (1/2\pi)\log(1/r(\boldsymbol{x}, \boldsymbol{s}))$ for two-dimensional potential problems, $r(\boldsymbol{x}, \boldsymbol{s})$ is the distance between source points $\boldsymbol{s}$ and collocation points $\boldsymbol{x}$, $\{\alpha_j\}$ the unknown coefficients to be determined. Substituting Eqs. (3) and (4) into boundary conditions (2), we obtain a matrix system

$$A\boldsymbol{\alpha} = \boldsymbol{b}, \tag{5}$$

where

$$A_{ij} = \begin{cases} G(\boldsymbol{x}_i, \mathbf{s}_j), & \boldsymbol{x}_i \in \Gamma_D \\ \frac{\partial G(\boldsymbol{x}_i, \mathbf{s}_j)}{\partial \mathbf{n}}, & \boldsymbol{x}_i \in \Gamma_N \end{cases}, \quad b_i = \begin{cases} h_D(\boldsymbol{x}_i), & \boldsymbol{x}_i \in \Gamma_D \\ h_N(\boldsymbol{x}_i), & \boldsymbol{x}_i \in \Gamma_N \end{cases}. \tag{6}$$

After determining the coefficients $\{\alpha_j\}$ with solving matrix system (5), we obtain the potential and flux with Eqs. (3) and (4). In the MFS, source points are collocated on an auxiliary boundary outside the computational domain (see Fig. 1(a) and (b)) to

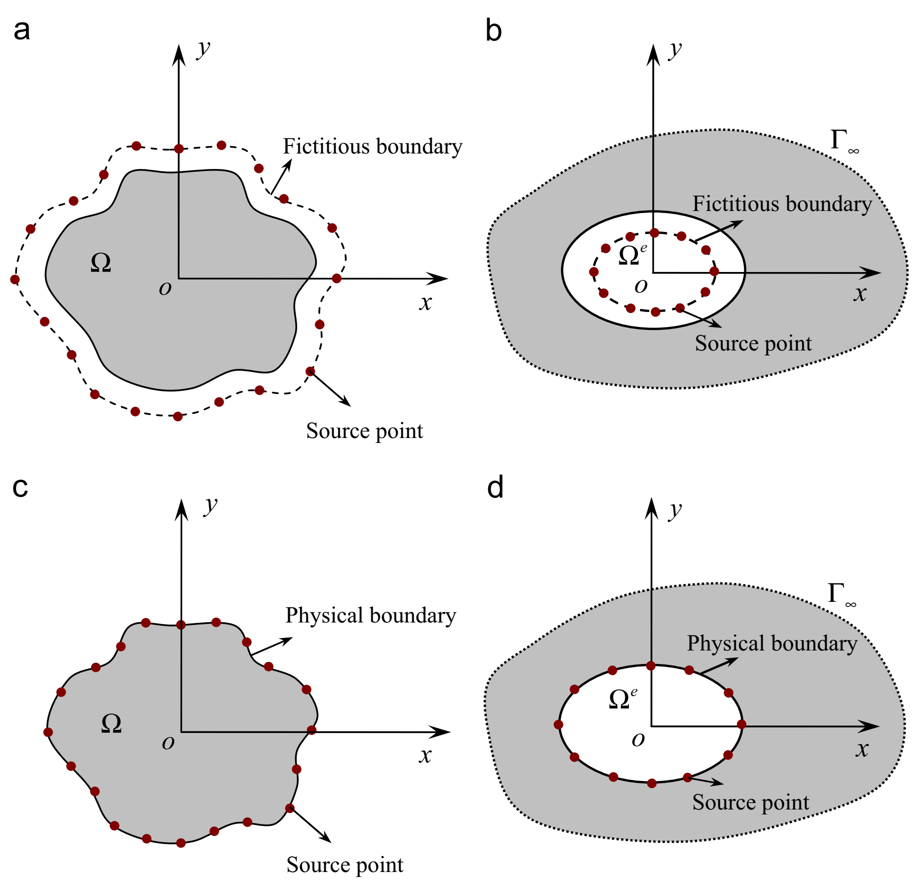

Fig. 1. Computational domain: (a) interior problems for MFS; (b) exterior problems for MFS; (c) interior problems for SBM; and (d) exterior problems for SBM.