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A three-dimensional implementation of the boundary element and level set based structural optimisation

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ABSTRACT

This paper presents a three-dimensional structural optimisation approach based on the boundary element and level set methods. The structural geometry is implicitly represented with the level set method, which evolves an initial structural model towards an optimal configuration using an evolutionary structural optimisation approach. The boundary movements in the three-dimensional level set based optimisation method allow automatic hole nucleation through the intersection of two surfaces moving towards each other. This suggests that perturbing only the boundary can give rise to changes not only in shape, but also in topology. At each optimisation iteration, the Marching Cubes algorithm is used to extract the modified geometry (i.e. the zero level set contours) in the form of a triangular mesh. As the boundary element method is based on a boundary discretisation approach, the extracted geometry (in the form of a triangular mesh) can be directly analysed within it. However, some mesh smoothing is required; HC-Laplacian smoothing is a useful algorithm that overcomes the volumetric loss associated with simpler algorithms. This eliminates the need for an additional discretisation tool and provides a natural link between the implicitly represented geometry and its structural model throughout the optimisation process. A complete algorithm is proposed and tested for the boundary element and level set methods based topology optimisation in three-dimensions. Optimal geometries compare well against those in the literature for a range of benchmark examples.

1. Introduction

The level set method (LSM) is an efficient numerical technique originally developed by Osher and Sethian [1] for the tracking of propagating interfaces with natural adaptation to topological changes such as merging and breaking. There is a wide variety of applications, including structural optimisation, in which the LSM has been successfully employed. Sethian and Wiegmann [2] first presented a level set (LS) based structural optimisation method, where shape and topological changes were accomplished through a von Mises stress based criterion. Osher and Santosa [3] proposed a LS based method using shape sensitivity analysis for the optimisation of an inhomogeneous drum for the frequency response. Wang et al. [4] proposed a shape sensitivity approach for the solution of minimum compliance problems. Allaire et al. [5] independently proposed a LS based optimisation method based on shape sensitivities for the solution of two and three-dimensional optimisation problems with both linear and non-linear structural material.

In the LS based optimisation approaches, the selection of an effective structural performance measuring tool, and an efficient optimisation technique, plays an important role for the solution of the optimisation problems. The performance measuring tool evaluates the structural response against the applied load and boundary conditions. These responses are then converted into a useful form by the optimisation technique, which evolves the structural geometry accordingly. The performance of a candidate design can be measured through a geometry mapping technique, which projects the implicitly represented geometry onto the structural model. The most commonly used geometry mapping techniques in the LS based structural optimisation are material distribution (density based), immersed boundary and conforming discretisation [6].

Most of the LS based optimisation methods utilise a fixed Eulerian type mesh with an “Ersatz material” approach [5] as an alternative finite element (FE) analysis tool. The structural geometry is represented through a density distribution function, i.e. $(\eta < \rho < 1)$, similar to the density based optimisation approach [7]. Solid material is represented by $(\rho = 1)$ and holes in the structure are replaced by a specified minimum relative density $(\rho = \eta)$. Wang et al. [4] and Allaire et al. [5] initially implemented the density based approaches in their proposed LS based topology optimisation methods. Although the fixed grid is a simple approach, it is not effective to capture the exact geometry of the boundary [5] and a highly dense grid distribution is always

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required near the boundary for high accuracy [8]. In addition, the presence of intermediate material densities along the structural boundary can result in indistinct boundary representation [9], which can cause uncertainty when transferring the optimum design to manufacture. A smoothed Heaviside function approach has been adopted to smooth the discontinuity at the boundary [10,11]. However, the numerical integration of the stiffness matrix may be less accurate [12].

The immersed boundary approach uses a non-body conforming fixed grid, where the structural geometry is not aligned with the Eulerian grid and can intersect some grid cells. This approach allows a clear boundary representation and avoids intermediate density material [6]. Sethian and Wiegmann [2] used the immersed interface method within a finite difference framework for the solution of the LS based topology optimisation problems. The extended finite element method (X-FEM) can also be used to evaluate the structural response at the design boundary through the local enrichment of elements intersected by the zero level set contour [13]. Belytschko et al. [14] combined the implicit boundary representation with the X-FEM approach for the solution of topology optimisation problems. Further implementations of the X-FEM within a level set framework can be found in [15,16]. Yamasaki et al. [9] developed a two-dimensional topology optimisation method for minimum compliance problems based on the immersed boundary mapping, boundary element and level set methods. The common problem reported in the implementation of the immersed boundary methods is the occurrence of small intersection of finite elements [15] or short boundary elements [9] while discretising the structural model. This can profoundly affect the accuracy of the structural response. Further, the use of immersed boundary techniques requires sophisticated codes and can make their implementation difficult and time consuming [6].

In contrast to the density and immersed boundary mapping, some of the LS based optimisation methods use two types of discretisation during the numerical implementation, i.e. a fixed Eulerian discretisation which maintains the LS function throughout the optimisation process, and a body conforming discretisation which exactly fits the design domain. Two different approaches can be used to discretise the design domain; the FEM based domain discretisation, and the boundary element method (BEM) based boundary only discretisation. The body conforming discretisation provides the most accurate analysis of the structural model, especially along the boundary. Ha and Cho [17] utilised an unstructured conforming discretisation approach for the optimisation of geometrically nonlinear structures within the LS framework. Yamasaki et al. [18] presented a boundary tracking approach for the LS based topology optimisation using a conforming discretisation approach and a geometry based re-initialisation scheme [19]. The use of BEM for the solution of minimum compliance problems within a two-dimensional LSM based optimisation method was first proposed by Abe et al. [20]. Later on, the proposed approach has also been extended for shape optimisation related to sound scattering problems [21].

In comparison with the immersed boundary mapping, the body conforming approach is attractive due to its simplicity and higher accuracy. A FEM based body conforming mapping may require special care to mesh a two-dimensional geometry and can make the discretisation of a three-dimensional arbitrary geometry more complicated and time consuming. As a consequence it could be difficult to ensure the analysis accuracy for a continuously changing finite element model. In contrast, the BEM based body conforming mapping is very attractive because it requires discretisation only at the design boundary, i.e. directly along the zero level set contours and significantly decreases the degrees of freedom in comparison with the FEM. This reduction of problem dimensionality simplifies considerably the re-meshing task (especially in three-dimensions), which can be performed efficiently and robustly. Thus, its rapid and robust re-meshing and accurate boundary solutions make the boundary based body mapping method a natural choice for the solution of the LS based shape and topology optimisation problems.

In a LS based optimisation method, an improvement in the design is mainly governed by changes in its shape. These changes can be carried out either with shape sensitivity information (e.g. [3,4,22–24]) or through an evolutionary approach based on the von Mises stress criterion (e.g. [2]). The sensitivity based techniques are popular because they are efficient although they require the computation of suitably accurate gradients, which may not be available. Moreover, these methods can often have difficulties in dealing with local optima. They are complex algorithms that are difficult to implement efficiently. Compared to the shape sensitivity approach, the evolutionary structural optimisation (ESO) methods are simple to implement, robust, and capable of dealing with almost any kind of structural optimisation problem, see for example [25]. The ESO schemes have remained popular on account of their simplicity and extensive empirical evidence of the fact that their optimal solutions closely resemble those derived by more rigorous descent methods (e.g. Li et al. [26]).

The use of ESO in a BEM and LSM based optimisation method has been first investigated in [27,28] for the solution of two dimensional optimisation problems. The implementation of a hole insertion mechanism in those studies provides optimal configurations insensitive to initial designs. However, the computation of the structural response at points inside the design domain is necessary to find the optimal locations for new hole insertions, and a direct extension of the proposed approach to three-dimensions would require additional efforts to calculate the structural response within the design domain. Instead, the boundary movements in a three-dimensional LS based optimisation method allow automatic hole nucleation through the intersection of two approaching surfaces [5], and consequently, the boundary only perturbation can give rise to changes not only in shape, but also in topology. Moreover, the BEM allows the evaluation of the structural response directly at the design boundary and its integration with the LSM, effectively handling shape and topology optimisation at the same time and eliminating the need for calculating the structural response within the design domain. This suggests a considerable reduction of the problem dimensionality in a three-dimensional implementation.

In a three-dimensional LS based optimisation approach, the structural geometry can be easily re-constructed in the form of a triangular surface mesh using a Marching Cubes (MC) algorithm. This allows automatic boundary discretisation of the modified structural geometry at each optimisation iteration. The accuracy and convergence of the boundary element analysis (BEA) for this discretised geometry can be further improved with mesh smoothing schemes, e.g. HC-Laplacian smoothing.

In the literature of LS based optimisation methods, the use of BEM is in the very early stages, and relatively few methods are available, e.g. [9,20]. In addition, these methods are limited to the solution of two-dimensional problems. The boundary-only intrinsic characteristic of the BEM together with the LSM makes this combination especially attractive for solving optimisation problems in three-dimensions, and requires a comprehensive investigation to propose an effective and reliable methodology. Therefore, the goal of the research work presented in this paper is to propose an optimisation approach for efficient utilisation of the advantageous features resulting from the integration of BEM, LSM and ESO. The authors have demonstrated this effective combination in two-dimensions [27,28] and, the extension of these ideas to three-dimensions in the current work. In comparison with competing FE-based approaches, it benefits from more