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Reliability analysis of Reissner plate bending problems by stochastic spline fictitious boundary element method

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ABSTRACT

In this paper the stochastic spline fictitious boundary element method (SFBEM) is presented for reliability analysis of Reissner plate bending problems in conjunction with the first-order reliability method (FORM). As a modified method for the conventional indirect boundary element method, SFBEM has been proved to be accurate and efficient in deterministic analyses. For the purpose of structural reliability analysis, SFBEM is introduced during the iteration process performed in the FORM to obtain the required values of structural responses and their derivatives with respect to the random variables considered. In particular, the gradient formulation for the Reissner plate bending problem has been derived using SFBEM in the current study. The present approach is validated with several numerical examples and a good agreement with solutions of the Monte Carlo simulation is observed.

1. Introduction

With an increasing demand for structural design, much work has been devoted to the problem of structural reliability. Many solution schemes of structural reliability have been developed in the framework of the finite element method (FEM) for its wide utilization in the deterministic region [1]. As compared with FEM, boundary element method (BEM) has emerged as a powerful tool for structural analysis for its unique advantages, for example, the discretization of the boundary that leads to significantly smaller systems of equations and the use of fundamental solutions for the infinite media that leads to a higher accuracy. This indicates that BEM could be used as an alternative numerical method for probabilistic structural analysis. Stochastic BEM has been successfully applied in solving stochastic elastostatic problems [2–4], stochastic elastodynamic problems [5], stochastic wave motion problems [6], stochastic potential problems [7], stochastic heat conduction problems [8], and stochastic groundwater flow problems [9], etc. Although much research has been devoted to BEM for stochastic analysis of various problems, little attention has been paid to the application of BEM to structural reliability analysis. Recently, a stochastic BEM has been proposed by the authors for reliability analysis of plane elasticity problems [10] and linear-elastic cracked structures [11].

The SFBEM is a modified indirect boundary element method. In SFBEM, nonsingular integral equations are derived using the fictitious boundary techniques, and spline functions with excellent performance are adopted as the trial functions to the unknown fictitious loads. Then the boundary-segment-least-square technique is employed for eliminating the boundary residues, which leads to the numerical solution to the integral equations. Because of these modifications, SFBEM is of high accuracy and efficiency in general. SFBEM was first applied to the solution of static plane elasticity problems [12], and so far it has been extended to multi-domain plane problems [13], orthotropic plane problems [14], plate bending problems [15], elastic fracture problems [16,17], stochastic elastostatic problems [4], and structural reliability problems [10,11].

In this paper, following the similar framework used in Ref. [10], a stochastic SFBEM is presented for reliability analysis of Reissner plate bending problems in conjunction with the FORM. SFBEM is incorporated into the iteration process of the FORM to calculate the required structural responses and their derivatives with respect to the random variables considered. The use of SFBEM in the formulation of the FORM makes it unnecessary to construct an explicit expression to the implicit limit state function of the problem, leading to a higher efficiency and better accuracy. The present approach is validated by comparing calculated solutions with those of Monte Carlo simulation for a number of example problems and a good agreement of the results is achieved.

2. SFBEM for analysis of Reissner plate bending problems

The Reissner plate bending theory is applicable to the analysis of thick plate bending problems since it can consider transverse shear deformation of the plate. In this theory, three boundary

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conditions rather than two boundary conditions as used in the Kirchhoff thin plate theory are adopted, which can reflect the real boundary conditions better.

Consider an elastic plate domain with I subdomains, each of which is of uniform property and thickness and is supported by elastic foundation. Let the boundary of the ith subdomain Ω_i be L_i, and suppose the body forces and the stiffness coefficient of foundation within Ω_i are F_i^l and $k_i (i=1,2,\ldots,I;\quad l=1,2,3)$, respectively, as shown in Fig. 1. Using the Winkler model, the reaction of the foundation can be expressed as $-k_i w$ with w being the deflection of the plate. Embed Ω_i into an infinite plate domain with the same property and thickness as in Ω_i, and apply unknown fictitious loads X_i^l $(l=1,2,3)$ along a fictitious boundary S_i outside Ω_i, whose shape is similar to that of the real boundary L_i, as also shown in Fig. 1. Then, under the combined action of the body forces F_i^l, the foundation reaction $-k_i w$ and the fictitious loads X_i^l, the components of displacement and internal force at any point P_0 in the infinite domain corresponding to Ω_i are as follows:

$$R(P_0)=\sum_{l=1}^{3}\int_{S_i}G_{R,i}^l(P_0;Q)X_i^l(Q)\mathrm{d}s+\sum_{l=1}^{3}\iint_{\Omega_i}G_{R,i}^l(P_0;Q_0)F_i^l(Q_0)\mathrm{d}\Omega$$
$$-\iint_{\Omega_i}G_{R,i}^1(P_0;Q_0)k_i(Q_0)w(Q_0)\mathrm{d}\Omega$$
$$(i=1,2,\ldots,I) \quad (1)$$

where $Q\in S_i$; $Q_0\in\Omega_i$; $R=w,\theta_x,\theta_y,Q_x,Q_y,M_x,M_y$ and M_{xy}; and $G_{R,i}^l-$ functions are fundamental solutions of Reissner plate bending problems [18].

Substituting Eq. (1) into the homogeneous boundary conditions along L_i $(i=1,2,\ldots,I)$, one has

$$\sum_{l=1}^{3}\int_{S_i}G_{k,i}^l(P,Q)X_i^l(Q)\mathrm{d}s+\sum_{l=1}^{3}\iint_{\Omega_i}G_{k,i}^l(P,Q_0)F_i^l(Q_0)\mathrm{d}\Omega$$
$$-\iint_{\Omega_i}G_{k,i}^1(P,Q_0)k_i(Q_0)w(Q_0)d\Omega=0$$
$$(i=1,2,\ldots,I;\quad k=1,2,3) \quad (2)$$

where $P\in L_i$ and k is for the three boundary conditions along L_i for Reissner plate bending problems; and $G_{k,i}^l$ are the kernel functions depending on the fundamental solutions and boundary conditions.

Dividing the domain into I subdomains, one may assume that there are J common boundaries and the jth boundary Γ_j is the boundary between the j_1th and j_2th subdomains $(j=1,2,\ldots,J;\ j_1,j_2\in[1,2,\ldots,I])$. Substituting Eq. (1) into the continuity and equilibrium conditions along Γ_j, one has

$$\sum_{l=1}^{3}\int_{S_{j_1}}g_{k,j_1}^l(p;Q_1)X_{j_1}^l(Q_1)\mathrm{d}s+\sum_{l=1}^{3}\iint_{\Omega_{j_1}}g_{k,j_1}^l(p;Q_{01})F_{j_1}^l(Q_{01})\mathrm{d}\Omega$$
$$-\iint_{\Omega_{j_1}}g_{k,j_1}^1(p;Q_{01})k_{j_1}(Q_{01})w(Q_{01})\mathrm{d}\Omega$$
$$=\sum_{l=1}^{3}\int_{S_{j_2}}g_{k,j_2}^l(p;Q_2)X_{j_2}^l(Q_2)\mathrm{d}s+\sum_{l=1}^{3}\iint_{\Omega_{j_2}}g_{k,j_2}^l(p;Q_{02})F_{j_2}^l(Q_{02})\mathrm{d}\Omega$$
$$-\iint_{\Omega_{j_2}}g_{k,j_2}^1(p;Q_{02})k_{j_2}(Q_{02})w(Q_{02})\mathrm{d}\Omega$$
$$(j=1,2,\ldots,J;k=1,2,\ldots,6) \quad (3)$$

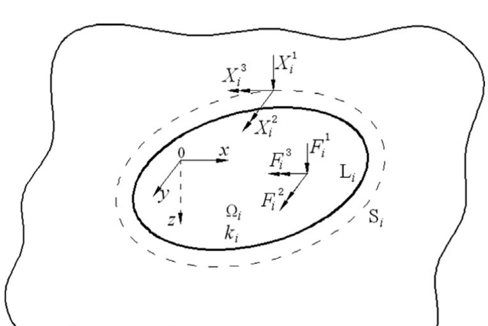

Fig. 1. The ith subdomain and its fictitious boundary.

where $p\in\Gamma_j$; $Q_1\in S_{j_1}$; $Q_2\in S_{j_2}$; $Q_{01}\in\Omega_{j_1}$; $Q_{02}\in\Omega_{j_2}$; and k is for the three continuity conditions and three equilibrium conditions along Γ_j; and g_{k,j_1}^l and g_{k,j_2}^l are kernel functions depending on the fundamental solutions and the continuity and equilibrium conditions.

Eqs. (2) and (3) are nonsingular fictitious boundary integral equations because the source points will never coincide with the field points in the kernel functions. These equations can be solved numerically. Expressing the unknown fictitious load functions X_i^l in terms of a set of B-spline functions and letting the integrations of the residues along each segment on boundary L_i and common boundary Γ_j equal zero, one can get

$$[G_i]\{X_i\}+\{B_i\}-[K_i]\{w_i\}=\{0\}\quad(i=1,2,\ldots,I) \quad (4)$$

for Eq. (2) and

$$[g_{j_1}]\{X_{j_1}\}+\{b_{j_1}\}-[k_{j_1}]\{w_{j_1}\}=[g_{j_2}]\{X_{j_2}\}+\{b_{j_2}\}-[k_{j_2}]\{w_{j_2}\}\quad(j=1,2,\ldots,J) \quad (5)$$

for Eq. (3), where $\{X_i\}$, $\{X_{j_1}\}$ and $\{X_{j_2}\}$ denote the column matrices consisting of the unknown spline node parameters of the fictitious loads along S_i, S_{j_1} and S_{j_2}; $[G_i]$, $[g_{j_1}]$ and $[g_{j_2}]$ denote the influence matrices of $\{X_i\}$, $\{X_{j_1}\}$ and $\{X_{j_2}\}$, respectively; $\{w_i\}$, $\{w_{j_1}\}$ and $\{w_{j_2}\}$ denote the column matrices consisting of the unknown deflections at different points within Ω_i, Ω_{j_1} and Ω_{j_2}; $[K_i]$, $[k_{j_1}]$ and $[k_{j_2}]$ denote the influence matrices of $\{w_i\}$, $\{w_{j_1}\}$ and $\{w_{j_2}\}$, respectively; $\{B_i\}$, $\{b_{j_1}\}$ and $\{b_{j_2}\}$ denote the known column matrices depending on the body forces within Ω_i, Ω_{j_1} and Ω_{j_2}.

According to the numbering of the subdomains and the common boundaries considered, all equations in Eqs. (4) and (5) can be combined into one global equation system as

$$[G]\{X\}+\{B\}-[K]\{w\}=\{0\} \quad (6)$$

where $\{X\}=[\{X_1\}^T\{X_2\}^T\cdots\{X_I\}^T]^T$; $\{w\}=[\{w_1\}^T\{w_2\}^T\cdots\{w_I\}^T]^T$; $[G]$ is dependent on $[G_i]$ $(i=1,2,\ldots,I)$ and $[g_{j_1}]$ and $[g_{j_2}]$ $(j=1,2,\ldots,J)$ in Eqs. (4) and (5), respectively; $\{B\}$ is dependent on $\{B_i\}$ $(i=1,2,\ldots,I)$ and $\{b_{j_1}\}$ and $\{b_{j_2}\}$ $(j=1,2,\ldots,J)$ in Eqs. (4) and (5), respectively; $[K]$ is dependent on $[K_i]$ $(i=1,2,\ldots,I)$ and $[k_{j_1}]$ and $[k_{j_2}]$ $(j=1,2,\ldots,J)$ in Eqs. (4) and (5), respectively.

There are two unknown column matrices $\{X\}$ and $\{w\}$ in Eq. (6). Therefore, an additional equation system should be supplemented to solve the problem. Using Eq. (1) and considering the deflection responses, one can easily obtain

$$\{w\}=[G_w]\{X\}+\{B_w\}-[K_w]\{w\} \quad (7)$$

where $[G_w]$ and $[K_w]$ are the influence matrices corresponding to $\{w\}$; and $\{B_w\}$ is the known column matrix due to the body forces. From Eq. (7), we have

$$\{w\}=[\tilde{K}]^{-1}([G_w]\{X\}+\{B_w\}) \quad (8)$$

where

$$[\tilde{K}]=[I]+[K_w] \quad (9)$$

and $[I]$ denotes the unit matrix with the same order as that of $[K_w]$.

Substituting Eq. (8) into Eq. (6) yields

$$[A]\{X\}+\{C\}=\{0\} \quad (10)$$

where

$$\left.\begin{array}{l}[A]=[G]-[K][\tilde{K}]^{-1}[G_w]\\ \{C\}=\{B\}-[K][\tilde{K}]^{-1}\{B_w\}\end{array}\right\} \quad (11)$$

Usually Eq. (10) needs to be solved on a least-squares basis as generally overdeterminate collocation is conducted to get a better solution at fewer fictitious boundary elements. The least-squares-based solution to Eq. (10) is [13]

$$\{X\}=-[A]^{+}\{C\} \quad (12)$$

where

$$[A]^{+}=([A]^T[A])^{-1}[A]^T \quad (13)$$