Engineering Analysis with Boundary Elements 51 (2015) 146–155

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

RBF-based meshless method for large deflection of elastic thin plates on nonlinear foundations

Mohammed M. Hussein Al-Tholaia, Husain Jubran Al-Gahtani *

Civil & Environmental Department Engineering, KFUPM, Dhahran, Saudi Arabia

ARTICLE INFO

Article history:
Received 28 May 2014
Received in revised form
30 October 2014
Accepted 30 October 2014
Available online 1 December 2014

Keywords:
Thin plate
Large deflection
Meshless
Movable edge
Immovable edge
Nonlinear foundation Winkler and
Pasternak

ABSTRACT

A simple, yet efficient method for the analysis of thin plates resting on nonlinear foundations and undergoing large deflection is presented. The method is based on collocation with the multiquadric radial basis function. In order to address the in-plane edge conditions, two formulations, namely w–F and u–v–w are considered for the movable and immovable edge conditions, respectively. The resulted coupled nonlinear equations for the two cases are solved using an incremental-iterative procedure. Three foundation models are considered, namely Winkler, nonlinear Winkler and Pasternak. The accuracy and efficiency of the method is verified through several numerical examples.

1. Introduction

The problem of interaction between structural foundations and supporting soil is of fundamental importance foundation design and therefore, it has attracted the attention of many researchers and engineers. The interaction is often represented by the classical problem of plate on elastic foundation. The main difficulty in the modeling of a plate on an elastic foundation is the determination of the contact pressure. The problem becomes more difficult if the plate is undergoing large deformation. The governing equations become coupled and higly nonlinear [1]. The available analytical methods are based on simplified assumptions and are limited to simple loading and boundary conditions and foundation models [1–8]. For such complicated problems, numerical methods offer convinient and reliable solutions. The ideal numerical method for the solution of nonlinear partial differential equations (PDEs) such as the one considered here should be high-order accurate, flexible with respect to the geometry, computationally efficient, and easy to implement. The conventional numerical methods that are commonly used usually fulfill one or two of the above criteria, but not all. Finite difference methods (FDM), finite element methods (FEM) and boundary element methods (BEM) have been the dominating methods for the numerical solution of PDEs [9–13]. Referring to the most dominant approach, i.e. FEM, it is highly flexible, but it is hard to achieve high-order accuracy and both coding and mesh generation become increasingly difficult as the problem dimension increases. The use of a mesh implies that specific procedures have to be devised just to define the mesh. Also, and to keep the order of the local approximation within reasonable limits, the element size has to be reduced, whenever better approximations are pursued. The extraordinary amount of work, which has been put into FEM research since its early years, has, one way or another, circumvented these and other problems associated with the existence of a mesh and made FEM the dominant approach for most problems in computational mechanics. Accordingly, many sophisticated powerful codes (e.g. ANSYS, ABACUS, COMSOL, etc.) have been established and have proven to be reliable in solving almost any computational mechanics problem. FDM can be made high-order accurate in resolving PDEs, but require a structured grid (or a collection of structured grids), which makes it difficult to model features of irregular domain. Furthermore, solutions of PDEs using FDM can be derived from the assumptions of the local interpolation schemes and require a mesh to support the localized approximations, however, the construction of a mesh in two or more dimensions is a non-trivial problem. In recent years, BEM has become a powerful alternative to FEM and FDM, especially for problems involving high gradients and stress concentrations. It has been successfully applied to solve the problems of large deflection of thin elastic plates. However, this was possible by devising some techniques to overcome the inherent deficiency of BEM as a self-standing numerical method in handling nonlinearities.

Nevertheless, the possibility of obtaining numerical solutions for PDFs without resorting to element frame, that is meshless technique,

* Corresponding author.

http://dx.doi.org/10.1016/j.enganabound.2014.10.011
0955-7997/

has been the goal of many researchers throughout the computational mechanics community for the past two decades or so. Radial Basis Function (RBF)-based collocation method, as one of the most recently developed numerical techniques, so-called mesh free or meshless methods, has attracted attention in recent years especially in the area of computational mechanics [14–19]. This method does not require mesh generation which makes them advantageous for nonlinear problems that require frequent re-meshing such as the ones considered in this study. The roots of RBF go back to the early 1970s when it was first used for fitting scattered data [20]. In the early 1980s, it was coupled with BEM in a technique called dual reciprocity-boundary element method where the RBF was employed to transform the domain integrals into boundary integrals [21]. Thereafter, many researchers have used RBF in conjunction with BEM to solve various problems in computational mechanics. The method, however, has not been applied directly to solve partial differential equations until 1990 by Kansa [23,24]. Since then, many researchers have suggested several variations to the original method [25].

In this paper, a multi-quadric (MQ)-RBF-based meshless model is developed for the solution of large deflection of thin plates resting on nonlinear foundations. The model is capable of handling different plate boundary conditions and foundations models. The accuracy of the model is validated through several numerical examples.

2. Governing equations

2.1. w–F formulation

The governing equations for large deflection of plates can be expressed in terms of the deflection w and a stress function F [1]

$$\nabla^4 w = \frac{q}{D} - \frac{P}{D} + \frac{t}{D} NL(w,F), \tag{1}$$

$$\nabla^4 F = -\frac{E}{2} NL(w,w) \tag{2}$$

where q is the distributed load, t is the plate thickness and $D=(Et^3)/(12(1-\upsilon^2))$ is the flexural rigidity of the plate having elastic constants E and υ , P is the foundation interaction force to be defined in Section 2.3, $NL(w,F)$ is a nonlinear differential operator given by

$$NL(w,F) = \left[\left(\frac{\partial^2 F}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) + \left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) - 2\left(\frac{\partial^2 F}{\partial x \partial y}\right)\left(\frac{\partial^2 w}{\partial x \partial y}\right)\right] \tag{3}$$

and

$NL(w,w)$ is obtained by replacing F by w in the foregoing equation. The stress function F is related to the memberane forces N_x, N_y, and N_{xy} by the following differential operators:

$$N_x = t\left(\frac{\partial^2 F}{\partial y^2}\right), \quad N_y = t\left(\frac{\partial^2 F}{\partial x^2}\right), \quad N_{xy} = -t\left(\frac{\partial^2 F}{\partial x \partial y}\right) \tag{4}$$

The bending moments M_x, M_y, and M_{xy} are related to w by the following differential operators respectively:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}\right), \quad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \upsilon\frac{\partial^2 w}{\partial x^2}\right),$$
$$M_{xy} = D(1-\upsilon)\left(\frac{\partial^2 w}{\partial x \partial y}\right) \tag{5}$$

The equivalent transverse shear forces V_x, V_y, are given by

$$V_x = Q_x - \frac{\partial M_{xy}}{\partial y}, \quad V_y = Q_y - \frac{\partial M_{xy}}{\partial x};$$

$$Q_x = -D\frac{\partial}{\partial x}(\nabla w), \quad Q_y = -D\frac{\partial}{\partial y}(\nabla w) \tag{6}$$

The resultant bending moment and equivalent shear force on the boundary are given by

$$M_n = -D\left\{\upsilon\nabla^2 w + (1-\upsilon)\left(n_x^2\frac{\partial^2 w}{\partial x^2} + n_y^2\frac{\partial^2 w}{\partial y^2} + 2n_x n_y\frac{\partial^2 w}{\partial x \partial y}\right)\right\} \tag{7}$$

$$V_n = -D\left\{ (n_y(1-n_x^2(\upsilon-1)))\frac{\partial^3 w}{\partial y^3} + \left(n_x(1-n_y^2(\upsilon-1))\right)\frac{\partial^3 w}{\partial y^3} + n_x\left(-2n_x^2(\upsilon-1) + n_y^2(\upsilon-1) + \upsilon\right)\frac{\partial^3 w}{\partial y^2 \partial x} + n_y\left(n_x^2(\upsilon-1) - 2n_y^2(\upsilon-1) + \upsilon\right)\frac{\partial^3 w}{\partial x^2 \partial y}\right\} \tag{8}$$

where n_x and n_y are the x and y components of the unit vector normal to the boundary.

The general boundary conditions for large deflection of plates can be classified into following two types:

1) Transverse boundary conditions which are encountered in both small and large deflection formulations. For this type, we will consider that at each boundary point there are two prescribed boundary conditions

   (a) $BC_{w1}(w)=0$ where $BC_{w1}(w)=w$ or $BC_{w1}(w)=V_n$ (9)

   (b) $BC_{w2}(w)=0$ where $BC_{w2}(w)=\frac{\partial w}{\partial n}$ or $BC_{w2}(w)=M_n$ (10)

2) In-plane boundary conditions which have to be addressed in the case of large deflection formulation. For a movable edge, the inplane boundary conditions are given by

$$BC_{F1}(F) = BC_{F2}(F) = 0 \quad \text{where } BC_{F1}(F) = F \text{ and } BC_{F2}(F) = \frac{\partial F}{\partial n} \tag{11}$$

The w–F formulation given above was based on the formulation presented in [29] and the condition for the movable edge case was $F = dF/dn = 0$, according to [29]. The stress function was not used to analyze problems with an immovable edge. The governing equations written in terms of the three displacements components u, v and w are used to analyze problems with immovable edges, as discussed in the following sections.

2.2. u–v–w Formulation

The details of the derivation can be found in the appendix. For briefety, we present only the final equations including the foundation interaction forces

$$L_{11}(u) + L_{12}(v) = NL_1(w), \tag{12}$$

$$L_{21}(u) + L_{22}(v) = NL_2(w) \tag{13}$$

$$\nabla^4 w = \frac{q}{D} - \frac{p}{D} + NL_3(u,v,w) \tag{14}$$

where

$$L_{11} = \frac{2\partial_{xx} + (1-\upsilon)\partial_{yy}}{2(1-\upsilon^2)}, \quad L_{12} = L_{21} = \frac{2\partial_{xy}}{2(1-\upsilon)}, \quad L_{22} = \frac{2\partial_{yy} + (1-\upsilon)\partial_{xx}}{2(1-\upsilon^2)} \tag{15}$$

$$NL_1(w) = -\frac{(1+\upsilon)w_{xy}w_y + w_x(2w_{xx} + (1-\upsilon)w_{yy})}{2(1-\upsilon^2)} \tag{16}$$

$$NL_2(w) = -\frac{(1+\upsilon)w_{xy}w_x + w_y(2w_{yy} + (1-\upsilon)w_{xx})}{2(1-\upsilon^2)} \tag{17}$$