Engineering Analysis with Boundary Elements 54 (2015) 68–75

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Dynamic analysis of a laterally loaded pile in a transversely isotropic multilayered half-space

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ARTICLE INFO

Article history:
Received 14 October 2014
Received in revised form
8 January 2015
Accepted 11 January 2015
Available online 17 February 2015

Keywords:
Dynamic response
Laterally loaded pile
Transversely isotropy
Multilayered half-space
FEM–BEM coupling

ABSTRACT

A finite element and indirect boundary element coupling method is presented for the time-harmonic response of a laterally loaded floating pile embedded in a transversely isotropic multilayered half-space. The floating pile is modeled as a Bernoulli-Euler beam using the finite element method (FEM), while the soil is modeled by using an indirect boundary element method (BEM) based on the fundamental solution for a transversely isotropic multilayered half-space. Then the governing equation of the interaction between the pile and transversely isotropic multilayered half-space is deduced by coupling FEM and BEM. Numerical examples are performed to validate the presented theory and to investigate the impact degree of anisotropy and layering arrangement on the dynamic response of a pile.

1. Introduction

Piles embedded in continuous media subjected to lateral harmonic loads have widespread application value in several branches of engineering. The piles are usually modeled as beams and solved by numerous methods, while the fundamental solution of soil is obtained by solving the governing differential equations of elastic medium. Then the pile-soil interaction problem is solved based on the continuity conditions between piles and soil. The above interaction problem was first presented by Tajimi [1] in 1969, who investigated the dynamic response of a single pile embedded in an elastic stratum. Since then, many researchers [2–16] have made extensive studies on the dynamic interaction between a pile or pile group and elastic isotropic media. Among these references, Pak and Jennings [8] presented a rigorous solution for the dynamic response of a single pile in an elastic half-space under transverse excitations, where the interaction problem was formulated as a Fredholm integral equation of the second kind. Kaynia and Kausel [10] used the Green's functions for layered media to analyze the dynamic stiffnesses of piles and pile groups in a layered half-space, and their works were usually considered to be the standard solutions for the dynamic response of piles and pile groups in viscoelastic media. On the other hand, some numerical techniques such as FEM [12], BEM [13,14] and FEM–BEM coupling [15,16] were applied to tackle this problem. Kuhlemeyer [12] used the finite element method to cope with the interaction problem of static and dynamic laterally loaded piles embedded in elastic media. With the aid of the fundamental solution for a periodic point force in a half-space, Sen et al. [13] put forward a boundary element method for the three-dimensional steady-state analysis of piles and pile groups in homogeneous soils. Mamoon et al. [14] introduced two boundary element methods to evaluate the impedance and compliance functions of piles and inclined pile groups embedded in a homogeneous soil medium. Based on a BEM-FEM coupling model, Padrón et al. [15] and Millán and Domínguez [16] obtained the solutions for vertical or horizontal time-harmonic response of piles and pile groups in viscoelastic or poroelastic soils.

As can be seen from the above references, the soil is usually treated as isotropic or layered isotropic media and the property of the anisotropy is neglected. Published papers with the consideration of pile–soil interaction in the transversely isotropic medium can be seldom found in the literature. The pioneering work was presented by Liu and Novak [17], who used the finite element method in the piles combined with dynamic stiffness matrix of soils to investigate the dynamic impedance of piles in transversely isotropic layered media. However, the subsequent studies mainly concentrated on the interaction among rigid piles [18,19]. Recently, Gharahi et al. [20] extended the solution of Pak and Jennings [8] to a transversely isotropic elastic half-space. As we know, natural soils are usually layered due to the long-term sedimentation process, and have different properties in the horizontal direction from those of the vertical direction [21–23]. Furthermore, the behavior of lateral vibration of piles is closely related to the soil properties, especially the horizontal direction properties. Therefore, it is absolutely essential to study the

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http://dx.doi.org/10.1016/j.enganabound.2015.01.008
0955-7997/

dynamic behavior of laterally loaded piles embedded in transversely isotropic multilayered media. However, the solutions for this topic have been rarely reported so far.

In this paper, the dynamic response analysis of a floating pile embedded in a transversely isotropic multilayered half-space subjected to horizontal harmonic excitations is performed by using a finite element and indirect boundary element coupling method. According to the Bernoulli–Euler beam theory, the floating pile is treated as a one-dimensional structure and discretized by applying a three-node element of FEM, whereas the multilayered half-space is treated as a three-dimensional elastic continuum modeled by means of an indirect BEM, whose kennel function is based on the dynamic solution for a transversely isotropic multilayered half-space [24]. Then, the pile–soil interaction problem is solved in virtue of the coupling between FEM and BEM. Finally, numerical examples are performed to compare with the existing exact solution for an isotropic half-space to confirm the accuracy of the proposed method and to study the influence of the characters of transversely isotropy and layering arrangement on the dynamic response of a pile.

2. FE equations for the pile

The general equation of motion for the pile in the absence of internal damping is as follow [25]

$$\mathbf{M}\ddot{\mathbf{u}}^p(t)+\mathbf{K}\mathbf{u}^p(t)=\mathbf{F}(t) \tag{1}$$

where $\mathbf{M}$ is the mass matrix of the pile; $\mathbf{K}$ is the stiffness matrix of the pile; $\mathbf{u}^p(t)$ is the vector of nodal displacements and rotations, and $\ddot{\mathbf{u}}^p(t)$ is the second derivation with respect to t of $\mathbf{u}^p(t)$; $\mathbf{F}(t)$ is the vector of nodal forces.

In this paper, the piles are subjected to time-harmonic load with circular frequency ω, thus the vectors of nodal displacements together with nodal forces are expressed in the following forms

$$\mathbf{u}^p(t)=\mathbf{u}^p e^{i\omega t} \tag{2a}$$

$$\mathbf{F}(t)=\mathbf{F}e^{i\omega t} \tag{2b}$$

where $\mathbf{u}^p$ and $\mathbf{F}$ are the amplitudes of $\mathbf{u}^p(t)$ and $\mathbf{F}(t)$, respectively.

With the elimination of the harmonic time factor $e^{i\omega t}$, substituting Eqs. (2a) and (2b) into Eq. (1) yields

$$(\mathbf{K}-\mathbf{M}\omega^2)\mathbf{u}^p=\mathbf{F} \tag{3}$$

The pile is modeled as a one-dimensional bar and discretized by a three-node beam element with 5 node parameters, of which three are lateral displacements (u_k, u_l, u_m) and two are rotations (θ_k, θ_m) (see Fig. 1). The lateral displacement along the element is approximated by a set of fourth degree shape functions as [15]

$$u=\varphi_1 u_k+\varphi_2\theta_k+\varphi_3 u_l+\varphi_4 u_m+\varphi_5\theta_m \tag{4}$$

where

$$\varphi_1=-\tfrac{3}{4}\zeta+\zeta^2+\tfrac{1}{4}\zeta^3-\tfrac{1}{2}\zeta^4 \tag{5a}$$

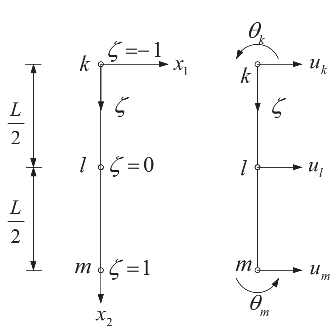

Fig. 1. Finite element definition for the pile.

$$\varphi_2=-\tfrac{1}{8}\zeta L+\tfrac{1}{8}\zeta^2 L+\tfrac{1}{8}\zeta^3 L-\tfrac{1}{8}\zeta^4 L \tag{5b}$$

$$\varphi_3=1-2\zeta^2+\zeta^4 \tag{5c}$$

$$\varphi_4=\tfrac{3}{4}\zeta+\zeta^2-\tfrac{1}{4}\zeta^3-\tfrac{1}{2}\zeta^4 \tag{5d}$$

$$\varphi_5=-\tfrac{1}{8}\zeta L-\tfrac{1}{8}\zeta^2 L+\tfrac{1}{8}\zeta^3 L+\tfrac{1}{8}\zeta^4 L \tag{5e}$$

in which L is the element length; ζ is the elemental dimensionless coordinate and $\zeta=(2x_2-L)/L, (-1\le\zeta\le 1)$.

With the aid of the principle of virtual displacements and the shape functions above, the consistent mass matrix and the stiffness matrix for an element can be obtained as [25]

$$\mathbf{M}^e=\frac{\rho_p AL}{1260}\begin{bmatrix}260 & 20L & 80 & -46 & 7L\\ 20L & 2L^2 & 8L & -7L & L^2\\ 80 & 8L & 512 & 80 & -8L\\ -46 & -7L & 80 & 260 & -20L\\ 7L & L^2 & -8L & -20L & 2L^2\end{bmatrix} \tag{6a}$$

$$\mathbf{K}^e=\frac{E_p I_p}{5L^3}\begin{bmatrix}316 & 94L & -512 & 196 & -34L\\ 94L & 36L^2 & -128L & 34L & -6L^2\\ -512 & -128L & 1024 & -512 & 128L\\ 196 & 34L & -512 & 316 & -94L\\ -34L & -6L^2 & 128L & -94L & 36L^2\end{bmatrix} \tag{6b}$$

where ρ_p is the density of the pile; A is the cross-sectional area of the pile; E_p is Young's modulus of the pile; I_p is the inertia moment of the section of the pile.

The nodal forces of the pile can be decomposed into two parts, which are the lateral force and bending moment applied at the pile head as well as the distributed forces along the pile-soil interface, i.e.

$$\mathbf{F}=\mathbf{F}^T+\mathbf{T}\mathbf{q}^p \tag{7}$$

where $\mathbf{F}^T$ is the lateral force and bending moment vector applied at the pile head; $\mathbf{q}^p$ is the nodal tractions vector along the pile-soil interface for the pile; $\mathbf{T}$ is global transform matrix of equivalent nodal forces of the pile.

As is illustrated in Fig. 2, the external forces defined over the element are schematized and the tractions along the pile-soil interface are approximated by a set of second degree shape functions as [15]

$$q_p=\phi_1 q_k+\phi_2 q_l+\phi_3 q_m \tag{8}$$

where

$$\phi_1=\tfrac{1}{2}\zeta(\zeta-1) \tag{9a}$$

$$\phi_2=1-\zeta^2 \tag{9b}$$

$$\phi_3=\tfrac{1}{2}\zeta(\zeta+1) \tag{9c}$$

Similarly, by using the principle of virtual displacements and the shape functions of Eqs. (5a)–(5e) and Eqs. (9a)–(9c), the transform matrix of equivalent nodal forces for an element can

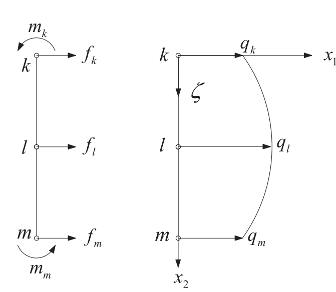

Fig. 2. Nodal forces and tractions along the pile–soil interface of an element.