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A local boundary integral method for two-dimensional particle-driven gravity currents simulation

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ABSTRACT

A meshless local boundary integral equation method (LBIEM) is used to analyse gravity currents flow in two-dimensional domains. The method solves the incompressible Navier–Stokes equations with a transport equation for the particle concentration. The characteristic-based split scheme is used to solve the governing equations. The LBIEM basic equations are derived via interpolation using radial basis functions. Two numerical test cases are presented here; both are focused on the typical lock-exchange channel flow. The LBIEM procedure produces stable solutions with results comparable to those of other conventional methods.

1. Introduction

Gravity currents increase when a heavier fluid propagates into a lighter one. This phenomena is frequently encountered in environment applications [1]. Particle-driven gravity currents create a special group of these flows when the difference in density is caused by the concentration of suspended particles. The particles may settle or become resuspended. For this reason these currents are known as non-conservative gravity currents [2]. Moreover, particles do not exactly follow the fluid flow. When the size of particles is small the velocity of particles can be expressed as a sum of the fluid velocity and a constant settling velocity (see, e.g. [3]).

To study the dynamics of particle-driven gravity currents a number of laboratory experiments were conducted. Most of them were focused on the spreading of particulate suspensions released into a rectangular channel full of clear fluid (see e.g. [4,5]). The direct numerical solution of this problem is usually based on a numerical scheme that employs a spatial discretization based on sine and cosine expansions in the horizontal direction together with finite differences in the vertical direction and a time discretization performed in a fully explicit manner with a third-order Runge–Kutta scheme. This scheme is very efficient but it is limited to rectangular areas (see [6,3,7]).

Meshless methods have recently been presented as attractive tools for solving computational fluid dynamics problems. They do not require the mesh generation employed by other methods, because they are defined by a cloud of points. Therefore, they are less computationally expensive than mesh-generation methods because the mesh generating phase is significantly reduced and because they increase flexibility in adapting the density of the nodal points at any place of the problem domain such that the resolution and fidelity of the solution can be improved easily. These features make meshless methods powerful tools in the solving of a wide range of scientific problems. During the past year, various meshless methods have been developed by different groups. These include smooth particle hydrodynamics (SPH) [8], the least square collocation meshless method [9–11], the meshless local Petrov–Galerkin (MLPG) method [12,13], the local boundary integral equation method (LBIEM) [14–16], and the radial basis integral equation method (RBIEM) [17–19]. Ghasemi et al. [20] used SPH to demonstrate the possibility of this method to solve density currents flow.

The present paper focuses on the LBIEM using the characteristic based split scheme (CBS) to solve the two-dimensional particle-driven currents flow. The radial basis functions (RBF) approximation is used for interpolating the variables. Two different test cases of 2D lock-exchange flows are solved and compared with previous solutions [6,21,7] to present the possibility of LBIEM as a solution for this type of problem.

2. Governing equations

The motion of the fluid phase is governed by the Navier–Stokes equations which can be written in its primitive variables as

$$\frac{\partial u_i}{\partial x_i} = 0$$

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$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_j}(u_j u_i)=\nu\frac{\partial^2 u_i}{\partial x_j \partial x_j}-\frac{1}{\rho}\frac{\partial p}{\partial x_i}-\frac{C}{\rho}F_i \tag{1}$$

where u_i is the fluid velocity vector component, p is the pressure, ν is the kinematic viscosity, ρ is the density of a liquid, C is the particle-number density (i.e. the concentration), and F_i represents Stoke's drag which is the dominant flow force on an individual particle in direction i [7]. The particle field can be treated in an Eulerian manner (see [7]) and is governed by the following advection–diffusion equation:

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial x_j \partial x_j}-\frac{\partial}{\partial x_i}(u_{pi}C) \tag{2}$$

where D is the coefficient of diffusion and u_{pi} is the particle velocity in the direction i. The particle velocity can be considered as (see also [23])

$$u_{pi}\approx u_i+u_{si}-\tau(1-\beta)\left[\frac{\partial}{\partial x_j}(u_j u_i)+\frac{\partial u_i}{\partial t}\right] \tag{3}$$

where u_i is the ith component of the fluid velocity, u_{si} is the ith component of the settling velocity of particles, τ is the particle response time, and β is the coefficient that depends on the ratio of the particle and fluid densities. When the particles are assumed to be spherical and sufficiently small the particle response time and the settling velocity can be written as

$$\tau=\frac{d^2(\rho_p/\rho+1/2)}{18\nu f},\quad \beta=\frac{3}{2(\rho_p/\rho+1)},$$

$$u_{si}=\tau(1-\beta)g_i=\frac{d^2(\rho_p-\rho)}{18\mu f}g_i \tag{4}$$

Here, d is the particle diameter, ρ_p is the particle density, f is the correction for non-Stokesian drag and g_i is the gravity vector component $\mathbf{g}=\{0,-g\}$ [2].

Eqs. (1) and (2) can be made dimensionless using transformations of coordinates and velocities

$$\tilde{x}_i=\frac{x_i}{L}\ \tilde{u}_i=\frac{u_i}{u_b} \tag{5}$$

where L is the characteristic length and u_b is the bulk velocity defined as [3]

$$u_b=\sqrt{\tilde{g}L}\ \tilde{g}=g\frac{\rho_1-\rho_0}{\rho_0} \tag{6}$$

The dimensionless concentration and pressure can be defined as

$$\tilde{C}=\frac{\rho-\rho_0}{\rho_1-\rho_0}\ \tilde{p}=\frac{p}{\rho_0 u_b^2} \tag{7}$$

The dimensionless governing equations can be written as

$$\frac{\partial \tilde{u}_i}{\partial x_i}=0$$

$$\frac{\partial \tilde{u}_i}{\partial t}+\frac{\partial}{\partial x_j}(\tilde{u}_j\tilde{u}_i)=-\frac{\partial \tilde{p}}{\partial x_i}+\frac{1}{\sqrt{Gr}}\frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j}-\tilde{C}e_i^g$$

$$\frac{\partial \tilde{C}}{\partial t}=\frac{1}{Sc\sqrt{Gr}}\frac{\partial^2 \tilde{C}}{\partial x_j \partial x_j}-\frac{\partial}{\partial x_i}\left(\tilde{u}_{pi}\tilde{C}\right) \tag{8}$$

where $e_i^g=(0,-1)$ is the unity vector pointing in the direction of gravity. Three dimensionless parameters appear in (8), namely the Grashof number Gr, the Schmidt number Sc, and the dimensionless particle velocity $\tilde{u}_{pi}$, which are defined as [7]

$$Gr=\left(\frac{u_b L}{\nu}\right)^2,\quad Sc=\frac{\nu}{D}$$

$$\tilde{u}_{pi}=\tilde{u}_i+\tilde{u}_{si}-\tilde{\tau}\left[\frac{\partial}{\partial x_j}(\tilde{u}_j\tilde{u}_i)+\frac{\partial \tilde{u}_i}{\partial t}\right] \tag{9}$$

where the particle Stokes number $\tilde{\tau}$ can be computed as

$$\tilde{\tau}=\frac{\tau(1-\beta)u_b}{L} \tag{10}$$

A characteristic-based split (CBS) algorithm is used to solve this problem (see [24–26]). The time derivative of the velocity in the second equation in (8) can be replaced with a difference resulting in the following relation:

$$\tilde{u}_i^{n+1}=\tilde{u}_i^n+\Delta t\left[\frac{1}{\sqrt{Gr}}\frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j}-\tilde{C}e_i^g-\frac{\partial}{\partial x_j}(\tilde{u}_j\tilde{u}_i)+\frac{\Delta t}{2}\tilde{u}_k\frac{\partial}{\partial x_k}\left(\frac{\partial}{\partial x_j}(\tilde{u}_j\tilde{u}_i)+\tilde{C}e_i^g\right)\right]^n-\Delta t\frac{\partial \tilde{p}^{n+1}}{\partial x_i} \tag{11}$$

where upper indexes n and $n+1$ indicate time steps and $\Delta t=t^{n+1}-t^n$ is the length of the time interval. The last term in the square brackets acts as the stabilizing term (see [27]). Eq. (11) is simplified using the fractional time step approximation (e.g., [10,28]), which computes the intermediate velocity $\tilde{u}_i^*$ using the simplified momentum equation

$$\tilde{u}_i^*=\tilde{u}_i^n+\Delta t\left[\frac{1}{\sqrt{Gr}}\frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j}-\tilde{C}e_i^g-\frac{\partial}{\partial x_j}(\tilde{u}_j\tilde{u}_i)+\frac{\Delta t}{2}\tilde{u}_k\frac{\partial}{\partial x_k}\left(\frac{\partial}{\partial x_j}(\tilde{u}_j\tilde{u}_i)+\tilde{C}e_i^g\right)\right]^n \tag{12}$$

Comparing (11) and (12) gives

$$\tilde{u}_i^{n+1}=\tilde{u}_i^*-\Delta t\frac{\partial \tilde{p}^{n+1}}{\partial x_i} \tag{13}$$

The intermediate velocity components $\tilde{u}_i^*$ do not satisfy the continuity equation in (8). The velocity components $\tilde{u}_i^{n+1}$ must satisfy the continuity equation, which implies

$$\frac{\partial}{\partial x_i}\left(\tilde{u}_i^*-\Delta t\frac{\partial \tilde{p}^{n+1}}{\partial x_i}\right)=0 \tag{14}$$

A pressure Poisson equation results directly from (14)

$$\frac{\partial^2 \tilde{p}^{n+1}}{\partial x_i \partial x_i}=\frac{1}{\Delta t}\frac{\partial \tilde{u}_i^*}{\partial x_i} \tag{15}$$

with Neumann boundary conditions that accrue from (13)

$$\left(\frac{\partial \tilde{p}}{\partial n}\right)^{n+1}=\frac{1}{\Delta t}\left(\tilde{u}_i^*-\tilde{u}_i^{n+1}\right)n_i \tag{16}$$

where n_i is the outer normal vector component in the i direction.

In the last step the dimensionless concentration $\tilde{C}$ can be solved using the CBS algorithm (see [24,29])

$$\tilde{C}^{n+1}=\tilde{C}_i^n+\Delta t\left[\frac{1}{Sc\sqrt{Gr}}\frac{\partial^2 \tilde{C}^n}{\partial x_j \partial x_j}-\frac{\partial}{\partial x_j}\left(\tilde{u}_{pj}^{n+1}\tilde{C}^n\right)+\frac{\Delta t}{2}\tilde{u}_{pk}^{n+1}\frac{\partial}{\partial x_k}\left(\frac{\partial}{\partial x_j}\left(\tilde{u}_{pj}^{n+1}\tilde{C}^n\right)\right)\right] \tag{17}$$

3. Meshless local integral formulation

The area of interest Ω with the boundary Γ is covered by points within the area and also on the global boundary (see Fig. 1). Consider a local circular sub-domain Ω_s with boundary Λ_s centered at every point s. This sub-domain is regular around all the internal points, but at the points on the global boundary this local boundary consists of a part of the global boundary intersected with the local sub-domain Γ_s (see Fig. 2). To express the local boundary integral form of the governing equations developed in the previous section in a domain Ω_s, we apply the weighting residual principle to Eqs. (12), (13), (15) and (18) to obtain the