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A regularized time-domain BIEM for transient elastodynamic crack analysis in piezoelectric solids

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ABSTRACT

A time domain non-hypersingular traction boundary integral equation method (BIEM) is proposed for dynamic crack analysis of piezoelectric solids. Using the boundary integral equation method, the time domain hypersingular integral equations for a dynamic crack in a 2D infinite piezoelectric solid subjected transient loads are derived. Considering the properties of the fundamental solutions, the hypersingular integral equations are reduced to singular integral equations by using the technique of integration by parts, in which the unknown functions are the tangential derivatives of the displacement and electrical potential discontinuities of the crack surfaces. To solve the time domain singular integral equations numerically, the quadrature formula of Lubich is applied for the temporal discretization, while the Gauss–Chebyshev quadrature method is used for the spatial discretization. Numerical examples are carried out to verify the accuracy of the present method by comparing the numerical results obtained by other scholars. Finally, several numerical results are presented and discussed to show the effects of the mechanical impact loading, crack-face conditions and piezoelectric coupling coefficient on the dynamic stress intensity factors.

1. Introduction

Owing to their abilities to convert energy between electrical and mechanical, piezoelectric materials are widely applied in transducers, actuators, sensors and so on. However, cracks and cracks-like defects exist in many brittle piezoelectric solids. Besides, these smart devices always work under dynamic loadings such as shock, sudden impact. Thus, dynamic fracture analysis is an important issue to evaluate the mechanical and electrical integrity, the reliability and durability of piezoelectric devices.

Since analytical solutions can be obtained only for very simple crack geometry and loading configurations, numerical methods are essential to analyze arbitrary crack and structure configurations. Enderlein et al. [1] presented an explicit finite element scheme to analyze plane cracks in piezoelectric structures. An extended finite element method was applied to simulation of stationary dynamic cracks in piezoelectric solids under impact loading by Bui and Zhang [2]. Liu and Dai [3] introduced a point interpolation mesh free method for static and mode frequency analysis two dimensional piezoelectric structures. Sladek et al. [4] proposed a meshless local Petrov–Galerkin method for plane piezoelectricity. Integral transform technique is also widely used in dynamic analysis. Chen and Yu [5–7] have studied the cracked piezoelectric solid under anti-plane impact. Shindo et al. [8] have studied the impact response of a finite crack in an orthotropic piezoelectric ceramic. Wang and Yu [9] have studied the response of piezoelectric strip with a crack subjected to the mechanical and electrical impacts. Moreover, the integral transform technique has also been applied into study of magneto-electro-elastic materials [10–17]. For the transform method, the numerical inversion of Laplace transform must be calculated carefully to obtain steady results.

BIEM and BEM are the most popular numerical methods in the solution of static and dynamic crack problems in piezoelectric solids. Dynamic fundamental solutions for frequency-domain have been presented in [18–19] and for time-domain have been presented in [20–22]. Due to dynamic piezoelectric fundamental solutions do not have closed form expressions and quite complex, it is the biggest challenge to form an easy and efficient numerical implementation. To circumvent this difficulty, dual reciprocity BEM has been developed in [23–25]. Frequency domain BEM has been presented by Denda et al. [18] for eigenvalue problems. Time harmonic crack analysis has been presented in [26–30] by using non-hypersingular traction BIEM to study the sensitivity of SIFs on the frequency of applied

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mechanical and electrical load. For time-domain transient dynamic crack analysis, Saez et al. [31], Garcia et al. [32,33] has proposed a time domain BEM using a combination of the strongly singular displacement boundary integral equations and the hypersingular traction boundary integral equations to transient dynamic analysis, and a time-domain collocation Galerkin BEM has been implemented by Wunshe et al. [34]. Especially, the convolution quadrature formula of Lubich [35,36] was firstly introduced into dynamic fracture analysis by Zhang [37], which is quite stable and accurate. To solve the Cauchy type singular integral equations, BEM was used in [26–30] and Gauss–Chebyshev method was used in [9,10,12].

In this paper, a time-domain singular BIEs for dynamic analysis of an infinite piezoelectric solid with stationary crack is presented. Impact loading acted upon the crack faces is considered, which the impact loading can be either mechanical, or electrical, or combination of both. As for the crack faces, electrically impermeable and permeable conditions are both taken into consideration. Due to the corresponding initial-boundary problem can't be solved analytically, the basic unknown functions which are the tangential derivatives of the extended displacements discontinuities across the crack faces have to be solved numerically. The Lubich's convolution quadrature formula is adopted to approximate the temporal convolution integral. And the classical Gauss–Chebyshev quadrature method is applied to deal with the spatial singular integral. After this numerical discretization, a system of linear algebraic equations is obtained. Meanwhile the single valued displacement conditions can be discretized as the same way. These simultaneous equations are solved step by step, and the transient dynamic stress and electrical displacement intensity factors are computed.

2. Basic equations

Under the quasi-electric assumption, and in the absence of body forces and electric charges, the motion equilibrium equations and the Gauss's law for the electric displacement can be expressed as [38]

$$\sigma_{ji,j} = \rho \ddot{u}_i; \quad D_{i,i} = 0 \tag{1}$$

where ρ is the mass density, u_i denotes displacements, σ_{ij} and D_i represent the mechanical tensor and the electric displacement vector respectively. Throughout the analysis, a comma after a quantity designates spatial derivative, and superscript dot stands for temporal derivative. The conventional summation rule over repeated indices is implied. The constitutive equations for homogeneous and linear piezoelectric materials are given by

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{ijk}E_k; \quad D_i = e_{kli}\varepsilon_{kl} + \gamma_{ik}E_k \tag{2}$$

where c_{ijkl} is the elasticity tensor, e_{ijk} is the piezoelectric tensor, γ_{ik} is the dielectric tensor. The relations between strain tensor ε_{ij} and displacement u_i as well as the electric field vector E_i and the scalar electric potential φ are given by

$$\varepsilon_{ij} = \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right); \quad E_i = -\varphi_{,i} \tag{3}$$

For convenience, the generalized displacements, the generalized stresses and the generalized elasticity are defined as follow:

$$U_I = \begin{cases} u_I, & I = 1,2 \\ \phi, & I = 4 \end{cases}$$

$$\Sigma_{iJ} = \begin{cases} \sigma_{iJ}, & J = 1,2 \\ D_i, & J = 4 \end{cases} \quad E_{iJKl} = \begin{cases} c_{iJKl}, & J,K = 1,2 \\ e_{Kli}, & J = 4, K = 1,2 \\ e_{Jil}, & K = 4, J = 1,2 \\ -\gamma_{il}, & J = K = 4 \end{cases} \tag{4}$$

Thus, the generalized motion equations and generalized constitutive equations are written as

$$\Sigma_{iJ,i} = \rho\delta^*_{JK}\ddot{U}_K; \quad \Sigma_{iJ} = E_{iJKl}U_{K,l} \tag{5}$$

where the generalized Kroneker delta $\delta^*_{JK}$is defined as

$$\delta^*_{JK} = \begin{cases} \delta_{JK}, & J,K = 1,2 \\ 0, & otherwise \end{cases} \tag{6}$$

lower case latin indices take the values 1 and 2(elastic), while capital latin indices take the values 1, 2(elastic) and 4(electric).

A crack in an infinite, two-dimensional piezoelectric solid is considered in this paper. The initial conditions at time$t \le 0$ are specified as

$$U_I(\mathbf{x},t) = \dot{U}_I(\mathbf{x},t) = 0 \tag{7}$$

As for crack-faces boundaries, the conditions of the electrically impermeable crack are prescribed

$$p_I(\mathbf{x},t) = p_I^*(\mathbf{x})H(t), \quad \text{for} \quad \mathbf{x} \in S_C. \tag{8a}$$

For electrically permeable crack, the electrical potential ϕ across the crack should satisfies

$$\tilde{U}_4(\mathbf{x},t) = \phi(\mathbf{x} \in S_{C^+}, t) - \phi(\mathbf{x} \in S_{C^-}, t) = 0. \tag{8b}$$

here, $p_I^*(\mathrm{x})$ represent the amplitude of the prescribed generalized crack-face loading, $H(t)$ stands for the Heaviside step function, S_{C^+} and S_{C^-} represent the upper and lower crack-faces.

3. Boundary integral equations

By using the Radon transform technique, the time-domain extended fundamental solutions have been analytically obtained by Wang [22]. The extended displacement fundamental solution U^G_{IJ} can be expressed as

$$U^G_{IJ}(\mathbf{x},\mathbf{y},t) = -\frac{H(t)}{8\pi^2}\int_{|\mathbf{n}|=1}\sum_{m=1}^{M}\frac{\Lambda^m_{IJ}}{\rho c_m^2}\frac{1}{c_m t + \mathbf{n}\cdot(\mathbf{x}-\mathbf{y})}d\mathbf{n} + \frac{1}{2\pi\sqrt{\Delta}}\log(R)\delta_{3I}\delta_{3J}\delta(t) \tag{9}$$

where $\mathbf{n} = (n_1, n_2)$, c_m and Λ^m_{IJ} denote the wave propagation vector, the phase velocities and the projection operator given in [22], $\mathbf{x}$ is the source point and $\mathbf{y}$ is the field point, Δ and R are defined as

$$\Delta = \mathbf{det}(\gamma_{ij}); \quad R = \sqrt{\gamma_{ij}^{-1}(x_i - y_i)(x_j - y_j)} \tag{10}$$

The traction fundamental solution T^G_{IJ} can be obtained by

$$T^G_{IJ}(\mathbf{x},\mathbf{y},t) = \textstyle\sum^G_{kIJ}(\mathbf{x},\mathbf{y},t)\cdot n_k(\mathbf{y}) \tag{11}$$

with

$$\textstyle\sum^G_{kIJ}(\mathbf{x},\mathbf{y},t) = E_{kIRl}\frac{\partial U^G_{JR}(\mathbf{x},\mathbf{y},t)}{\partial y_l} \tag{12}$$

where n_k ($\mathbf{y}$)are the components of the unit outward normal at the field point.

For an infinite cracked piezoelectric solid, the extended displacements at the source point can be obtained by integral formulation, in which the generalized Betti-Rayleigh reciprocity theorem is adopted

$$U_I(\mathbf{x},t) = -\int_{S_{C^+}} T^G_{IJ}(\mathbf{x},\mathbf{y},t)*\tilde{U}_J(\mathbf{y},t)ds(\mathbf{y}) \tag{13}$$