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# Interactive three-dimensional boundary element stress analysis of components in aircraft structures

T.M. Foster [a,*], M.S. Mohamed [b], J. Trevelyan [a], G. Coates [a], S.H. Spence [c], S.K. Walker [d]

[a] *School of Engineering and Computing Sciences, Durham University, Durham DH1 3LE, UK*
[b] *Institute for Infrastructure and Environment, Heriot-Watt University, Edinburgh EH14 4AS, UK*
[c] *Industrial Sponsor, BAE Systems (Operations) Limited, Samlesbury Aerodrome, Balderstone, Lancashire, BB2 7LF, UK*
[d] *Industrial Sponsor, Jesmond Engineering Limited, Brough Business Centre, Skillings Lane, Brough, East Yorkshire, HU15 1EN, UK*

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ABSTRACT

Computer aided design of mechanical components is an iterative process that often involves multiple stress analysis runs; this can be time consuming and expensive. Significant efficiency improvements can be made by increasing interactivity at the conceptual design stage. One approach is through real-time re-analysis of models with continuously updating geometry. Thus each run can benefit from an existing mesh and governing boundary element matrix that are similar to the updated geometry.

For small problems, amenable to real-time analysis, re-integration accounts for the majority of the re-analysis time. This paper assesses how efficiency can be achieved during re-integration through both algorithmic and hardware based methods. For models with fewer than 10,000 degrees of freedom, the proposed algorithm performs up to five times faster than a standard integration scheme. An additional six times speed is achieved on eight cores over the serial implementation. By combining this work with previously addressed meshing and solution schemes, real-time re-analysis becomes a reality for small three-dimensional problems. Significant acceleration of larger systems is also achieved.

This work demonstrates the viability of application in the aerospace industry where rapid validation of a range of similar models is an essential tool for optimising aircraft designs.


## 1. Introduction

Real-time analysis of two-dimensional models is now a reality [1]. However, real-time stress analysis of three-dimensional models presents numerous additional challenges. Various schemes based on the finite element method that aim to provide interactivity have been presented previously [2–5]. Meier et al. [6] review a range of deformable models that utilise both finite element and boundary element (BE) techniques. In this paper we address only BE implementations. The boundary element method (BEM) is a natural method to use where re-meshing is involved as changes need to be applied only to elements on the surface of the model, this enables the propagation of modifications to the mesh to be more easily contained within a small region. Wang et al. [7] present a BEM based scheme for interactive analysis for surgical simulation. However, this makes use of an extensive library of pre-computed solutions. In this work we aim to completely re-generate the boundary element system in real-time during each update iteration.

The primary aim of this work is to build a new stress analysis software package that is capable of accurately re-analysing three-dimensional computer models of mechanical components in real-time as the geometry is updated by the user. In this work, ‘Real-time’ refers to the re-analysis taking place within the refresh rate of the media on which the model is viewed. It is intended that this software should be used at the conceptual design stage to aid the engineer in optimising the design of components. Key to this is the identification of the interaction of feature stress concentrations. This makes it possible to design components more appropriately for their expected loading from the earliest design stages and will remove the need to make costly adjustments later in the design process if the geometry were found to be unsuitable. As the software is for conceptual purposes it should be intuitive to use and does not need to be able to handle the full range of complex geometry associated with the finished products but will often focus on sub regions of components.

The acceleration of boundary element algorithms has been the subject of a considerable volume of research over the last two

* Corresponding author. Tel.: +44 191 3342487; fax: +44 191 3342408.
*E-mail addresses:* timothy.foster@durham.ac.uk (T.M. Foster), m.s.mohamed@hw.ac.uk (M.S. Mohamed), jon.trevelyan@durham.ac.uk (J. Trevelyan), graham.coates@durham.ac.uk (G. Coates), stuart.spence@baesystems.com (S.H. Spence), simon.walker@jesmondengineering.com (S.K. Walker).

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decades, with most attention paid to the Fast Multipole Method (FMM) and Adaptive Cross Approximation (ACA) approaches. However, these approaches (in particular FMM) involve additional overheads that make them less attractive for the small problems that are the focus of the current work. The authors have studied ACA as a candidate approach for the real-time reanalysis, but find that it becomes cost-effective only for problem sizes larger than 5000 degrees of freedom. Although we show here that this is larger than the problem size offering real-time feedback, it is anticipated that it could very well be the most promising strategy quite soon.

To realise the goals of the current work in three dimensions, new boundary element techniques which incorporate a new re-meshing scheme and a modified re-integration algorithm, have been developed [8]. These new algorithms enable re-use of unchanged model geometry to limit propagation of changes through the mesh and thereby accelerate the reconstruction of the boundary element system of equations. The reconstruction of these equations is the subject of this paper as, for the small problems assessed in this work, the system construction time takes a significantly higher proportion of the total re-analysis time than re-solving the system; this can be seen in Figs. 22 and 23. The solution is carried out using a generalised minimal residual (GMRES) solver with full approximate LU preconditioning as developed by Foster et al. [8].

This work is being carried out with a view to applications in aerospace. The structures assessed in the aerospace industry cover a broad range of applications such as helicopters, civil airliners, missiles, training aircraft, military transport aircraft and compact fast jets. Whilst each of these structures may be subject to significantly different loading regimes and different life requirements, the fundamental critical design features are often of a broadly similar nature and an accurate assessment of their stress concentrations is equally vitally important in calculating fatigue life.

The two-dimensional BE software Concept Analyst [1], which makes use of many of the ideas presented in this paper but with a restriction to planar problems, has been used in the aerospace industry for several years where it has been found powerful in terms of its ability to rapidly assess a number of the types of critical features commonly encountered in aerospace structures. Extensive industrial testing has been carried out on the two-dimensional BE software, comparing results with those of both published sources and of fine-mesh finite element analyses. These comparisons have shown the BE software to be accurate, reliable, and extremely fast in comparison with other techniques. However, there have remained a number of key design features that could not be analysed with the two-dimensional software due to their three-dimensional nature, including problems such as a countersunk hole and interacting fillets. The current work to extend the capability of the software into the three-dimensional domain promises to open up several of these types of features to analysis whilst maintaining the established benefits of re-analysis based rapid BE analysis seen in two dimensions.

## 2. The boundary element method

The BEM is a standard method of analysis in the solution of partial differential equations, and is the subject of numerous texts including Becker [9]. This section contains a brief overview of the principal steps involved. We consider the problem of finding displacements and stresses in a linear elastic material comprising a domain $\Omega \subset \mathbb{R}^3$, having boundary $\partial\Omega = \Gamma$. We seek to solve the equations of linear elasticity subject to boundary conditions:

$$u_i(q) = \overline{u}, \quad q \in \Gamma_u \tag{1}$$

$$t_i(q) = \overline{t}, \quad q \in \Gamma_t \tag{2}$$

where $u_i$ and $t_i$ are displacement and traction components, respectively, $\overline{u}$ and $\overline{t}$ are prescribed displacement and traction boundary conditions, respectively, and $\Gamma = \Gamma_u \bigcup \Gamma_t$. In practice, the use of different boundary condition types in different coordinate directions at the same location is common, so that division of $\Gamma$ into separate Neumann and Dirichlet boundaries in this fashion is purely symbolic. A boundary integral equation (BIE) can be formulated for displacements at a source point, $p \in \Gamma$, due to tractions and displacements on $\Gamma$:

$$c_{ij}(p)u_i(p) + \int_\Gamma T_{ij}(p,q)u_i(q)\, d\Gamma(q) = \int_\Gamma U_{ij}(p,q)t_i(q)\, d\Gamma(q) \tag{3}$$

where $c(p)$ is a term introduced as a result of the limits applied to allow the strongly singular integral containing the traction kernel to be evaluated, so that the integral on the left hand side of Eq. (3) is evaluated in the Cauchy Principal Value sense. $T_{ij}$ and $U_{ij}$ form $3 \times 3$ matrices, $[T]$ and $[U]$, for each source-field point pairing and refer respectively to the traction and displacement kernels, given by

$$T_{ij} = \frac{-1}{8\pi(1-\nu)r^2}\frac{\partial r}{\partial n}\left[(1-2\nu)\delta_{ij} + 3\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}\right] + \frac{1-2\nu}{8\pi(1-\nu)r^2}\left[\frac{\partial r}{\partial x_j}n_i - \frac{\partial r}{\partial x_i}n_j\right] \tag{4}$$

$$U_{ij} = \frac{1}{16\pi(1-\nu)r}\left[(3-4\nu)\delta_{ij} + \frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}\right] \tag{5}$$

where $r = |q-p|$, $n_i$ and $n_j$ are components of the outward normal, $n$, at $q$, $\delta_{ij}$ is the Kronecker delta, $\mu$ is the shear modulus and $\nu$ is the Poisson's ratio of the material. The subscripts define directional components, so that $T_{ij}$ and $U_{ij}$ refer to a traction or displacement in the Cartesian direction $i$ at field point $q$, caused by a unit load in direction $j$ at the source point $p$. To solve the system numerically the boundary of the object must first be discretised into elements, forming a surface mesh. The discretised BIE can now be re-written in the local parametric element coordinates $(\xi, \eta)$:

$$\begin{aligned} c_{ij}(p)u_i(p) &+ \sum_{elem\ e}\int_{-1}^{1}\int_{-1}^{1} T_{ij}(p,q)N_k(\xi,\eta)J(\xi,\eta)\, d\xi\, d\eta u_i^{ke} \\ &= \sum_{elem\ e}\int_{-1}^{1}\int_{-1}^{1} U_{ij}(p,q)N_k(\xi,\eta)J(\xi,\eta)\, d\xi\, d\eta t_i^{ke} \end{aligned} \tag{6}$$

where $t_i^{ke}$ and $u_i^{ke}$ are the tractions and displacements, respectively, acting in the $i$ direction at node $k$ of element $e$. The vector $N$ contains the value of each shape function at the current integration point and $J$ is the Jacobian, which transforms the differential variables at the current point from the local into the global coordinate system. The vectors $u$ and $t$ are now the nodal displacements and tractions, respectively. For reasons of computational performance, we use the collocation form of the BEM requiring collocation of Eq. (6) at a sufficient number of points, $p$, that for convenience coincide with the nodal positions. Performing the integrations given in Eq. (6) a system of equations can be derived. These are given in matrix form as

$$[H]\{u\} = [G]\{t\} \tag{7}$$

where $[H]$ and $[G]$ contain terms resulting from the evaluation of the double integrals in (6). After applying boundary conditions, Eq. (7) can be rewritten in the form

$$[A]\{x\} = \{b\} \tag{8}$$

This system can now be solved as a set of linear equations to find the unknown tractions and displacements contained in $\{x\}$. Internal stresses may be found by declaring $p$ at the point of interest, substituting the now fully defined values of displacement