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A weighting-iteration method in the time domain for solving the scattering problem of a complex-shaped scatterer

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ABSTRACT

A weighting-iteration method in the time domain is developed to calculate the scattered waves from a complex-shaped scatterer. The incident waves can be mono-frequency or multi-frequency, and the complex object includes sharp edges and dramatic variations in geometry. The solid angles on the boundary elements of a complex-shaped scatterer are generally reduced to below the standard value of 0.5 for points on a smooth part of the boundary. These reduced solid angles destroy the convergence history during the iteration process in the time domain. A weighting function associated with the variation of solid angles is introduced to robust and rapid convergence in the time domain.

The new method is used to calculate the scattering from a cube with sharp edges and an indented surface. The weighting function speeds up the convergence history to reach a robust convergence for both mono- and multiple-frequency incident waves.

1. Introduction

The computation of scattered waves from an object is applied in many fields such as remote sensing and military detection. Scattering problems in underwater acoustics are an important topic of research, which depends on solving scattering problems accurately and efficiently.

Since the 1960s, researchers such as Schenck [7] and Burton and Miller [2] have attempted to resolve exterior scattering problems. Schenck [7] introduced the CHIEF method to overcome the problem of non-unique solutions at fictitious eigenfrequencies. Seybert et al. [15] developed a second-order boundary element method in the frequency domain, and Seybert and Soenarko [16] dealt with the infinite half-space problem. Seybert and Wu [17] and Li et al. [12] discussed arbitrary impedance on an infinite free surface. Chaosong [3] derived a direct boundary integral equation method in the frequency domain. Kress and Mohsen [11] and Ochmann [14] developed the source simulation technique in the frequency domain for faster computation of complex structures.

The time-domain method is a better choice for multi-frequency incident waves. Mansur and Brebbia [13], Groenbroom [6], Dohner et al. [5] and Araújo et al. [1] derived time-domain boundary element formulations for transient problems. According to Araújo et al. [1], numerical errors propagate forward in time and result in incorrect solutions. Kao and Kehr [10] proposed a robust iteration method in the time domain to solve scattered waves. Kao and Kehr [10] expressed the oscillatory integrals in the boundary integral equation relative to the retarded time, which made the iteration process easier. The iteration procedure proposed by Kao and Kehr [10] was proven to be convergent, and the results are therefore correct. Bi et al. [4] solved the transient acoustic radiation from an arbitrarily shaped source in the time domain using a cubic spline interpolation. Li et al. [9] presented a time-domain boundary integral equation framework to analyse the acoustic scattering from spherical objects in a homogeneous medium.

The solid angles of some of the panels on the surface of a complex-shaped scatterer will be clearly reduced and deviated from the standard value of 0.5, making it difficult to obtain convergent results in the time domain. Kao and Kehr [10] only solved objects with a standard solid angle value of 0.5, and thus did not consider the problem of deviating solid angles. In this paper, the method used by Kao and Kehr [10] is extended to solve the scattered waves from a complex object. A suitable weighting function relative to the solid angles is found, which corrects the convergence tendency and speeds up the convergence. Several iterations are initially conducted with this weighting function. The last result of these iterations is then treated as the initial condition for the next round of formal iterations without the weighting function. The selected object for the calculation features sharp edges and huge variations in geometry. Both mono- and multi-frequency incident waves are considered using the new method.

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2. Theoretical formulations

An arbitrarily shaped body immersed in a semi-infinite domain is considered, as shown in Fig. 1. The body surface is denoted by S, its outward normal by n, the scattered potential by φ_s, the incident potential by φ_i and the interface of the body and the infinite plane by Sc. The infinite plane of the interface can be simulated by a mirror-image body.

By applying Green's second identity and the Sommerfeld radiation condition, a boundary integral equation is defined on S,

$$C(P)\varphi_s(P) = \int_S \left(\varphi_s(Q)\frac{\partial G(P,Q)}{\partial n} - G(P,Q)\frac{\partial \varphi_s(Q)}{\partial n} \right) \mathrm{d}S. \tag{1}$$

$G(P,Q)$ is the half-space Green function that depends on both locations of a field point, P, and a source point, Q,

$$G(P,Q) = \frac{1}{4\pi}\left(\frac{e^{-ikr}}{r} + R_H \frac{e^{-ikr_i}}{r_i} \right), \tag{2}$$

where r is the distance between P and Q, and r_i is the distance between P and Q', the imaged point of Q, as shown in Fig. 1. The reflection coefficient, R_H, is equal to 1 for a rigid infinite plane, and -1 for a soft infinite plane. $C(P)$ in Eq. (1) is the solid angle for the field point, P, and can be evaluated as follows (see, e.g. Seybert and Wu [17]):

$$C(P) = 1 + \frac{1}{4\pi}\int_{S+S_c} \frac{\partial}{\partial n}\left[\frac{1}{r(P,Q)} \right] dS \tag{3}$$

if P is on S but not in contact with S_c, and

$$C(P) = \left(1 + R_H\right)\left\{ \frac{1}{2} + \frac{1}{4\pi}\int_{S+S_c} \frac{\partial}{\partial n}\left[\frac{1}{r(P,Q)} \right] dS \right\} \tag{4}$$

if P is on the intersection of S and S_c.

3. Formal iteration scheme

The iteration scheme for the scattered wave in the time domain proposed by Kao and Kehr [10] is

$$\begin{aligned} C(P)\varphi_s(P,t) &= \int_{-\infty}^{\infty}\int_S \varphi_s(Q)\cdot\frac{\partial G(P,Q)}{\partial n}\cdot e^{i\omega t} \mathrm{d}s \ \mathrm{d}\omega \\ &\quad - \int_{-\infty}^{\infty}\int_S G(P,Q)\cdot\frac{\partial \varphi_s(Q)}{\partial n}\cdot e^{i\omega t} ds \ d\omega \\ &= \mathrm{A}_{\mathrm{part}}(\varphi_s(P,t)) + B_{part}(\varphi_I(P,t)). \end{aligned} \tag{5}$$

A_{part} can be expressed in the time domain as

$$A_{part} = \sum_{m=1}^{M} \left\{ \begin{array}{c} \left[\varphi_s(Q,t-r/c)\cdot\left(\frac{-1}{r^2}\right) - \frac{\partial \varphi_s(Q,t-r/c)}{\partial t}\cdot\frac{1}{cr} \right]\cdot\left(\nabla\vec{r}.\ \vec{n}\right)+ \\ R_H\cdot\left[\varphi_s(Q,t-r_i/c)\cdot\left(\frac{-1}{r_i^2}\right) - \frac{\partial \varphi_s(Q,t-r_i/c)}{\partial t}\cdot\frac{1}{cr_i} \right]\cdot\left(\nabla\vec{r_i}.\ \vec{n_i}\right) \end{array} \right\}_m \cdot \frac{\Delta S_m}{4\pi}. \tag{6}$$

Because the solid boundary condition is applied and the incident potential is given, B_{part} in Eq. (5) is known.

$$\begin{aligned} B_{part} &= -\int_{-\infty}^{\infty}\int_S G(P,Q)\cdot\frac{\partial \varphi_s(Q)}{\partial n}\cdot e^{i\omega\ t}\ dS\ \ d\omega = \int_{-\infty}^{\infty}\int_S G(P,Q)\cdot\frac{\partial \varphi_I(Q)}{\partial n}\cdot e^{i\omega\ t}\ dS\ \ d\omega \\ &= \frac{1}{4\pi}\int_S\left[\left(\frac{1}{r}\right)\cdot\frac{\partial \varphi_I(Q,t-r/c)}{\partial n} \right] dS + \frac{R_H}{4\pi}\int_S\left[\left(\frac{1}{r_i}\right)\cdot\frac{\partial \varphi_I(Q,t-r_i/c)}{\partial n_i} \right] dS \\ &= \sum_{m=1}^{M}\left\{ \left[\nabla\varphi_I(Q,t-r/c)\cdot\vec{n} \right]\cdot\frac{1}{r} + R_H\cdot\left[\nabla\varphi_I(Q,t-r_i/c)\cdot\vec{n_i} \right]\cdot\frac{1}{r_i} \right\}_m \cdot\frac{\Delta S_m}{4\pi}. \end{aligned} \tag{7}$$

Please refer Kao and Kehr [10] for detailed derivations of A_{part} and B_{part}.

The oscillatory integrals in the boundary integral equation are expressed in forms relative to the retarded time, which makes the iteration process easier. Treating the incident wave as the sum of many harmonics, we calculate the time interpolation using a Fourier series. Therefore, the numerical error due to the time interpolation is minimised, and only the spatial discretisation is needed in the present method.

The iteration scheme is based on Eq. (5). B_{part} in Eq. (5) is known and calculated by the incident wave. An initial scattered potential, φ_s, on the body equal to zero is first prescribed. Then, the scattered potential field on the body on the left-hand side of Eq. (5) can be updated explicitly by calculating A_{part} according to Eq. (6).

The time step is repeated over the longest period ($T=1/f_{MIN}$) of the multi-frequency input. This iterative process is repeated until proper convergence is achieved. The time marching is divided into k time steps (ISTEP$=k$) within the longest period.

The procedure of the iteration scheme is indicated in Fig. 2 and further details can be found in Kao and Kehr [10].

4. Weighting-iteration method

Kao and Kehr [10] computed the scattered waves from a half sphere and a cubic object. From Kane [8] it is known that the value of a solid angle on a smooth surface is equal to 0.5. The solid angle of each panel on these two objects is almost 0.5 because there are no dramatic variations in their geometry.

If the scatterer is of a complex shape, the solid angles of some panels on the complex structure will be reduced and clearly deviate from the standard value of 0.5. This makes it difficult for the iteration scheme of Kao and Kehr [10] to find a convergent result and the convergence will be very slow. Therefore, in this paper, the iteration scheme in Eq. (5) is modified by introducing a weighting-iteration method in the time domain to solve the scattering problems of complex objects.

By reformulating Eq. (5), the iteration scheme can be re-presented as follows.

$$\begin{aligned} \varphi_s(P,t)_{n+1} &= \left(A_{part}\left(\varphi_s(P,t)_n\right) + B_{part}\left(\varphi_I(P,t)\right)\right)/C(P) \\ &= \varphi_s(P,t)_n + \Delta\varphi_s(P,t). \end{aligned} \tag{8}$$

The suffix $(\ )_{n+1}$ indicates the values by the $(n+1)$-th iteration, and $(\ )_n$ the values by the n-th iteration. The difference between the $(n+1)$-th and n-th iterations can be expressed as follows.

$$\Delta\varphi_s(P,t) = \left(A_{part}\left(\varphi_s(P,t)_n\right) + B_{part}\left(\varphi_I(P,t)\right)\right)/C(P) - \varphi_s(P,t)_n \tag{9}$$

To speed up the convergence, a weighting function is introduced to Eq. (8), as follows:

$$\varphi_s(P,t)_{n+1} = \varphi_s(P,t)_n + W.\ \Delta\varphi_s(P,t). \tag{10}$$

W is the weighting function for complex-shaped scatterers. Eq. (10) is only used for panels with deviating solid angles of less than 0.5. The original iteration scheme shown in Fig. 2 is thus

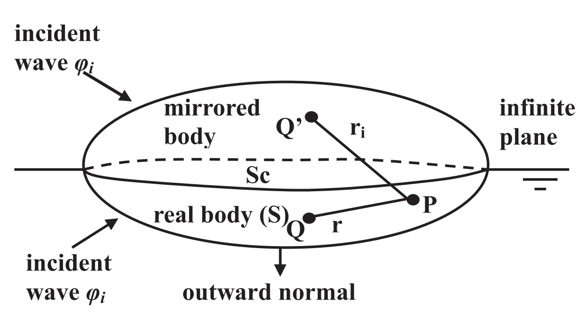

Fig. 1. A body of arbitrary shape on an infinite plane.