Engineering Analysis with Boundary Elements 62 (2016) 44–56

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Three efficient numerical models to analyse the step problem in shallow water

E.G.A. Costa [a,*], J.A.F. Santiago [a], L.M.C. Godinho [b], L.C. Wrobel [c], W.J. Mansur [a]

[a] COPPE/UFRJ–Program of Civil Engineering Federal University of Rio de Janeiro, CP 68506, CEP 21945-970 Rio de Janeiro, RJ, Brazil
[b] CICC–Department of Civil Engineering University of Coimbra, 3030-788 Coimbra, Portugal
[c] Brunel University London, Institute of Materials and Manufacturing, Uxbridge UB8 3PH, England

ARTICLE INFO

Article history:
Received 16 June 2015
Received in revised form
15 September 2015
Accepted 17 September 2015
Available online 22 October 2015

Keywords:
Boundary Element Method
Method of Fundamental Solutions
Green's functions
Ewald's method
Shallow water

ABSTRACT

In this paper, the problem of acoustic wave propagation in a waveguide of infinite extent is modelled, taking into account constant depth in each section of the sea. Efficient numerical strategies in the frequency domain are addressed to investigate two-dimensional acoustic wave propagation in a shallow water configuration, considering a step in the rigid bottom and a flat free surface. The time domain responses are obtained by means of an inverse Fast Fourier Transform (FFT) of results computed in the frequency domain. The numerical approaches used here are based on the Boundary Element Method (BEM) and the Method of Fundamental Solutions (MFS). In the numerical models only the inclined or vertical interface between the sub-regions of different depth are discretized, as Green's functions that take into account the presence of free and rigid surfaces are used. These Green's functions are obtained either by eigenfunction expansion or by Ewald's method. A detailed discussion on the performance of these formulations is carried out, with the aim of finding an efficient numerical formulation to solve the step problem in shallow water.

Contents

* Corresponding author.
E-mail address: edmundo_costa@coc.ufrj.br (E.G.A. Costa).

http://dx.doi.org/10.1016/j.enganabound.2015.09.005
0955-7997/

1. Introduction

Many analytical and numerical methods have been developed to simulate and analyse underwater acoustic wave propagation. The book by Jensen et al. [1] discusses in detail the different methodologies applied to solve the problem of wave propagation in acoustic environments that have interested many researchers over past decades. Some of the well-known methods are based on the acoustic ray theory [2], the normal modes method [3] and the parabolic equation [4].

A variety of numerical models has also been developed based on well-established approaches such as the finite difference, finite element and boundary element methods. Of these, the Boundary Element Method (BEM) permits an efficient analysis of underwater acoustic problems with complex shapes and complicated boundary conditions. The BEM has a number of advantages over other numerical methods [5], such as: it is very well suited for modelling homogeneous unbounded domains since it automatically satisfies the Sommerfeld radiation condition and thus involves a more compact description of the acoustic medium, requiring only the discretization of the problem boundaries, which considerably reduces the size of the final linear system of equations. However, the application of the boundary integral equation is often limited by the requirement of prior knowledge of the fundamental solutions and the appearance of singular or hyper-singular integrals in its formulation.

Another difficulty of the BEM in the analysis of acoustic wave propagation in shallow water occurs when more complex geometries are considered, requiring large discretization schemes. One way of avoiding this large discretization is by using Green's functions which directly satisfy the boundary conditions on the flat free surface and the rigid bottom of the ocean. Such Green's functions can be constructed using the image-source technique, but this leads to very slowly convergent series [6,7]. An alternative to improve the convergence of the series is to build a Green's function in the form of eigenfunction expansions, the so-called normal mode solution. This function is also an infinite series but if only the evanescent modes are considered and there are no propagating modes, the series becomes rapidly convergent owing to the exponentially decaying terms for the evanescent modes. In spite of that, the convergence problems of this series still remain when the source and the field points are positioned along the same vertical alignment [8,9].

Linton [10,11] and Papanicolaou [12] discuss mathematical techniques for accelerating slowly convergent series and show that Ewald's method is able to provide dramatic improvements in the speed of convergence, particularly when the source and field points are located along the same vertical line. This method has been successfully implemented in the boundary integral equation formulation by Venakides et al. [13], for the calculation of electromagnetic scattering of photonic crystals.

Santiago and Wrobel [8,9] discussed the implementation of Ewald's method in a BEM model for underwater acoustics. They compared the convergence of Ewald's method with that of eigenfunction expansions, showing a substantial reduction in the number of terms necessary for convergence of the series, particularly when the source and field points are positioned along the same vertical line. In the present paper, for the first time, Ewald's method is fully implemented in a BEM underwater acoustics model in which a vertical interface is discretized, significantly improving the performance of the method. The integration of the resulting singular integrals is also discussed in the paper.

In recent years, meshless methods have attracted great interest of scientists and researchers. The Method of Fundamental Solution (MFS) is one of these methods and it has been applied with success for scattering or radiation problems. Mathematically, the MFS is a very simple technique and it is also based on the prior knowledge of fundamental solutions, but not requiring the numerical and analytical integrations that need to be performed in the BEM. One disadvantage of the MFS is the determination of the position of the pseudo-boundary on which the singularities are placed. Karageorghis [14] has proposed a simple algorithm for estimating an optimal pseudo-boundary for certain boundary value problems. Costa et al. [15,16] have shown that, despite its simplicity, the MFS is a very interesting tool to efficiently predict wave acoustic propagation in shallow water.

In this paper, the Boundary Element Method and the Method of Fundamental Solutions are used to analyse, in the frequency domain, the two-dimensional acoustic wave propagation in a shallow water configuration, considering a step up on the bottom of the sea. Time domain signals are computed by means of an inverse fast Fourier transform of the numerical results in the frequency domain. Appropriate Green's functions are used limiting the number of discretized surfaces and consequently reducing the computational cost of the proposed models. These models are developed by using a sub-region technique, where only the inclined or vertical interface between the sub-regions of different depth has to be discretized. These Green's functions are obtained either by eigenfunction expansion or by Ewald's method. A set of numerical examples is performed in order to demonstrate the efficiency of the proposed models in the analysis of acoustic wave propagation in shallow water problems containing a step on the seabed. In addition, a detailed discussion on the performance of these formulations is carried out, with the aim of finding an efficient formulation to solve the acoustic step problem in shallow water in the frequency domain.

2. Governing equation of the problem

The problem of two-dimensional acoustic wave propagation in a region Ω of infinite extent in the longitudinal z-direction is analysed, taking into account the presence of a step up on the bottom of the sea, as shown in Fig. 1. If the velocity of sound is constant, the source of acoustic disturbance is time-harmonic and the medium in the absence of perturbations is quiescent, the problem is governed by the Helmholtz equation which can be written as:

$$\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = -Q\delta(\mathbf{x} - \boldsymbol{\xi}^s) \quad \text{in } \Omega \tag{1}$$

where $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$; $p(\mathbf{x})$ is the acoustic pressure; Q is the magnitude of the acoustic, sound-emitting source $\boldsymbol{\xi}^s$ located at (x_{ξ^s}, y_{ξ^s}); $\mathbf{x}$ is the observation point located at (x, y), $\delta(\mathbf{x} - \boldsymbol{\xi}^s)$ is the Dirac delta function, and $k = 2\pi f / c$ is the wave number, with f being the excitation frequency and c the sound propagation velocity.

The boundary conditions for the above described problem are given by:

- Dirichlet condition.

$$p(\mathbf{x}) = 0 \quad \text{in } \Gamma_F \tag{2}$$

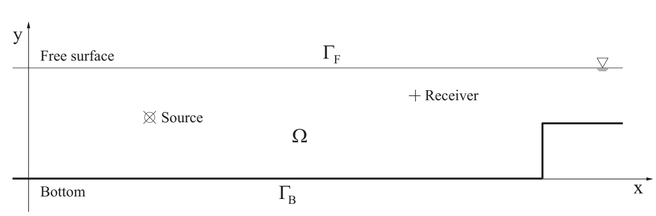

Fig. 1. Geometry of the problem.