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Dynamics of an air bubble induced by an adjacent oscillating bubble

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A B S T R A C T

This study is concerned with the collapse of an air bubble induced by its adjacent oscillating bubble, including the splitting of the air bubble and the subsequent transitions of the two split sub-bubbles from singly-connected to doubly-connected. The numerical modeling is based on the potential flow theory coupled with the boundary integral method. A two-vortex-rings model is put forward to further simulate the interaction between two toroidal bubbles and a singly-connected one, which is rarely seen in previous studies. To validate the numerical model, experiments are carried out for the dynamics of an air bubble induced by a spark generated bubble captured by a high speed camera. Our numerical results agree qualitatively with the experimental data. It is found that the strength parameter ε of the oscillating bubble greatly affects the jet velocity of air bubble and the length ratio l' of air bubble determines its collapsing pattern.

1. Introduction

The dynamics of bubbles has significant consequences for diverse applications, and it has become an important research field. Bubble dynamic behaviors may vary with the initial inner pressure, and therefore bubbles are classified according to their initial inner pressures in this study. When the initial inner pressure of a bubble is higher than that the ambient flow pressure, it would expand from its initial stage and thus is called an oscillating bubble in this study. Oscillating bubbles are found in a wide range of engineering applications, such as air gun bubbles for seabed exploration in the ocean engineering field [1] and underwater explosion bubbles in military [2]. When a bubble is in its equilibrium state, which means that its inner pressure, its surface tension and the ambient pressure are balanced, it would not oscillate by itself until the equilibrium state is disrupted and is called air bubble in this paper. Air bubbles are far more common, such as rising bubbles in water [3–6]. Many applications involve the interaction between an air bubble and an oscillating bubble, including anti-shock bubble curtain and oscillating bubble eliminating the wake. Besides, if a bubble oscillates near an air bubble, a jet can form threading the air bubble and reach a very high speed. This jet may have an effect on or even damage neighboring structures [7]. All above aroused the interest of worldwide researchers, however, there are few studies on dynamics of air bubble in proximity with an oscillating one. Chen et al. [8] carried out an experimental research on the evolution of an existing air bubble and another laser-induced expanding bubble in a thin ink sheet between two glass slices, and fragmentation of the air bubble was observed. Besides experimental study, the boundary integral method (BIM) based on potential flow theory is adopted for numerical study on the interaction between an air bubble and an oscillating bubble. The BIM has been verified repeatedly to simulate the motion of oscillating bubble [9–16] and predict the shape of rising bubbles [17–20]. Pain et al. [7] studied the jet in the initially stationary bubble induced by a spark-generated bubble both experimentally and numerically. They found that for increasing inter-bubble distance and increasing initial radius of the air bubble, jet velocity of the spherical air bubble decreases. And jet would not be formed inside the air bubble when the initial radius of the air bubble reaches a certain value. The interaction between an oscillating bubble and a non-oscillating air bubble attached a horizontal plate was investigated by Goh et al. [21]. The focus of their paper was to prevent a jet of oscillating bubble from being directed towards a nearby rigid boundary by attaching an approximately hemi-spherical air bubble onto the boundary.

In the previous works, a jet of the air bubble directed away from the oscillating bubble can be formed which penetrates the spherical air bubble and then the air bubble transits from a singly-connected form to a doubly-connected one [7]. However, simulation always stops at the very moment of the jet impact of air bubble and the interaction between a toroidal air bubble and an oscillating bubble is not investigated yet. The transition of air bubble to a toroidal one is an obvious barrier to the BIM. The flow

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domain becomes doubly connected after jet impact, which results in a non-unique solution problem for a potential flow model. Many works with the implementation of circulation model were performed for continued calculation of the toroidal bubble motion [11,17,22], among which a vortex ring model by Wang [23] is simpler and more widely used. In fact, an air bubble is neither quiescent nor spherical in a free field. In that way, the dynamics of air bubble induced by an oscillating bubble would change, which is the main concern in the present paper.

In this paper, the most basic case of the interaction between an oscillating bubble and an air bubble is studied both experimentally and numerically. We notice that an annular jet can form which splits the air bubble into two sub-bubbles. Then the two sub-bubbles would jet away and form two toroidal bubbles. The challenge for numerical simulation is that the fluid field transits from a doubly connected form to a triply connected one and no existing model could be used. Therefore, a two-vortex-rings model is built in this paper to further simulate the subsequent motion. In order to reveal the physics underlying the splitting formation process, the pressure field is calculated in this study. Normally, the Bernoulli equation in the potential flow theory is used to calculate the flow field pressure. However, it is difficult to calculate the term $\partial\varphi/\partial t$ in the Bernoulli equation, where φ is the velocity potential. A central differencing scheme is often used [24]. However, using the finite difference method to calculate $\partial\varphi/\partial t$ often results in unexpected errors, because the time steps used for simulating violently collapsing bubbles are very small. Therefore, we use the auxiliary function [25,26] to calculate $\partial\varphi/\partial t$ for accuracy.

This study deals with the dynamics of an air bubble induced by an adjacent oscillating bubble. Particular attention is focused on the effects of two dimensionless parameters (strength parameter of oscillating bubble and length ratio of air bubble) on air bubble shape evolution. The jet penetrating the air bubble can reach a high speed. As such, it is of special interest to investigate the relationship between the collapsing pattern and jet velocity of air bubble. In Section 2, a brief outline of the mathematical formulation based on incompressible potential flow theory and some numerical techniques are put forward, basic notation and definitions are also given in this section. In Section 3, a two-vortex-rings model is put forward to simulate the interaction between two toroidal bubbles and a singly-connected one, which is rarely seen in previous studies to the best knowledge of the authors. The numerical model is validated by both the convergence study and the comparison between the numerical and experimental results. In Section 4, the effects of the strength parameter of the oscillating bubble ε and the length ratio of the air bubble l' are investigated numerically with the initial volume of the air bubble and the inter-bubble distance in the experiment. Both the air bubble shape evolution and the jet velocity are calculated. And finally, conclusions are summarized.

2. Mathematical formulation and numerical techniques

2.1. Mathematical formulation

Consider the evolution of an air bubble induced by its adjacent oscillating bubble as shown in Fig. 1. This problem is axisymmetric along the z-axis where r is the radial coordinate. The oscillating bubble is assumed to have been initiated as a tiny high-pressure spherical bubble. The air bubble is assumed to be initially in the equilibrium state, with its inner pressure, surface tension and the pressure in the liquid balanced. The fluid domain outside the bubble is Ω. The initial distance between two bubble centers is D_{in}. In this paper, the axisymmetric cases considered with the origin of the coordinate system locating at the center of the oscillating bubble and z axis pointing upward.

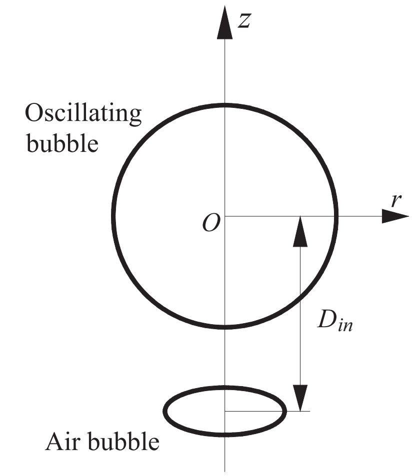

Fig. 1. Geometry and coordinate system used to model the dynamics of an air bubble induced by an adjacent oscillating bubble.

The fluid is assumed to be inviscid, incompressible and irrotational, for the reason that the duration of the interaction between air bubble and oscillating bubble is short and the Reynolds number is high. The velocity potential φ can be introduced into analysis which satisfies the boundary integral equation:

$$\vartheta\varphi(\boldsymbol{p}) = \iint_S \left(\frac{\partial\varphi(\boldsymbol{q})}{\partial n} \frac{1}{|\boldsymbol{r}(\boldsymbol{p}) - \boldsymbol{r}(\boldsymbol{q})|} - \varphi(\boldsymbol{q}) \frac{\partial}{\partial n} \left(\frac{1}{|\boldsymbol{r}(\boldsymbol{p}) - \boldsymbol{r}(\boldsymbol{q})|} \right) \right) dS \tag{1}$$

where $\boldsymbol{p}$ and $\boldsymbol{q}$ are the fixed point and the source point located at the boundary, ϑ is the solid angle at the fixed point $\boldsymbol{p}$ which equals 2π on a smooth surface, the normal derivative is defined as $\partial/\partial n = \boldsymbol{n} \cdot \nabla$ and $\boldsymbol{n}$ is the outward normal, and S is all the boundaries of the fluid domain.

The pressure inside the bubble is only related to its initial state which is assumed to obey the ideal gas law and the surface tension is taken into account because of the small scale of the bubbles. The pressure equilibrium on the bubble surface with the consideration of surface tension gives [11]:

$$P_l = P_c + P_0 \left(\frac{V_0}{V} \right)^{\lambda} - \sigma \cdot \kappa \tag{2}$$

where P_l is the liquid pressure, P_c is the constant vapor pressure, P_0 and V_0 are the initial pressure and volume of the bubble, respectively, λ is the ratio of the specific heats of the gas which is taken as $\lambda = 1.25$ for spark-generated bubble [2] and as $\lambda = 1.4$ for air bubble (ideal gas) [23], σ is the surface tension coefficient and κ is the local curvature of bubble surface.

Axisymmetric BIM is adopted to discretize Eq. (1) in this paper, and the local curvature of bubble surface can be obtained according to Young–Laplace equation:

$$\kappa = 1/R_1 + 1/R_2 \tag{3}$$

where R_1 and R_2 are radii of the principle curvatures along r and z axis, respectively. For axisymmetric model, arc length s can be seen as a variable for the function r- and z-coordinate, so parameter equations $r = r(s)$ and $z = z(s)$ are introduced. Formula of R_1 and R_2 can be obtained by taking the derivative of the parameter equations [27]:

$$\frac{1}{R_1} = \frac{r''(s)z'(s) - r'(s)z''(s)}{[r'^2(s) + z'^2(s)]^{3/2}} \tag{4a}$$

$$\frac{1}{R_2} = \frac{-z'(s)}{r(s)[r'^2(s) + z'^2(s)]^{1/2}} \tag{4b}$$