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A stable nodal integration method with strain gradient for static and dynamic analysis of solid mechanics

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ABSTRACT

A stable nodal integration method with strain gradient (SNIM-SG) for curing the temporal instability of node-based smoothed finite element method (NS-FEM) is proposed for dynamic problems using linear triangular and tetrahedron element. In each smoothing domain, except for considering the smoothed strain into the calculation of potential energy functional as NS-FEM, a term related to strain gradient is taken into account as a stabilization term. The proposed SNIM-SG can achieve appropriate system stiffness in strain energy between FEM and NS-FEM solutions and obtains quite favorable results in elastic and dynamic analysis. The accuracy and stability of SNIM-SG solution are studied through detailed analyzes of benchmark cases and practical engineering problems. In elastic-static analysis, it is found that SNIM-SG can provide higher accuracy in displacement field than the reference approaches do. In free vibration analysis, the spurious non-zero energy modes can be eliminated effectively owing to the fact that SNIM-SG solution strengths the original relatively soft NS-FEM, and SNIM-SG is confirmed to obtain fairly accurate natural frequency values in various examples. All in all, SNIM-SG cures the flaws of NS-FEM and enhances the dominant of nodal integration. Thus, the efficacy of the presented formulation in solving solid mechanics problems is well represented and clarified.

1. Introduction

Finite element method (FEM) and mesh-free methods have been widely developed to solve various types of practical problems of engineering and science.

Both mesh-free [1] and FEM methods can be classified collectively as a Galerkin method [2–4], a Petrov-Galerkin method [5], and other methods [6]. Gauss integration is commonly used in Galerkin methods for integration of weak form. Due to the complexity involved in Gauss integration for Galerkin methods, attempts have been made to develop nodal integration methods for computation [7–9].

In 1994, an element-free Galerkin method which is applicable to arbitrary shapes but requires only nodal data is applied to elasticity and heat conduction problems [7]. The paper shows the method does not exhibit any volumetric locking, the rate of convergence can exceed that of finite elements significantly and a high resolution of localized steep gradients can be achieved. Another advantage of the method is that it requires no postprocessing for the output of strains and stresses or other field variables which are derivatives of the primary-dependent variables since these quantities are already very smooth. The method offers tremendous potential in industrial application and in the implementation of adaptivity. However, this method leads to a numerical instability due to under integration of the weak form and vanishing derivatives of shape functions at the nodes, which definitely hinders it's further implementation and development [8].

A strain smoothing stabilization for nodal integration is proposed [8,10] to eliminate spatial instability in nodal integration. For convergence, an integration constraint (IC) is introduced as a necessary condition for a linear exactness in the mesh-free Galerkin approximation. The gradient matrix of strain smoothing is shown to satisfy IC using a divergence theorem. Results show that the accuracy and convergent rates in the mesh-free method with a direct nodal integration are improved considerably by the proposed stabilization conforming nodal integration method. And the proposed method can provide even better accuracy than Gauss integration for Galerkin mesh-free method. It was further generalized for accommodating discontinuous displacement functions, leading to the generalized smoothed Galerkin (GS-Galerkin) weak form [9] that forms the foundation of the node-based smoothed finite element method (NS-FEM) [11,12] and a class of numerical methods, such as the node-based smoothed point interpolation method (NS-PIM) [13], smoothed finite element

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method (SFEM) [14–20], edge-based smoothed finite element method (ES-FEM) [14] and so on. These solutions based on strain smoothing have been extended or developed in recent years, and have been successfully used to analyze linear and nonlinear solids [21–25], linear and nonlinear plates and shells [26], free and forced vibration problems [27,28], singular field problems [29–32], piezoelectric structures [33], composite plates [34,35], functionally graded material plates [36,37] and so on. The NS-FEM can be viewed as a variant model of FEM. It has very attractive properties

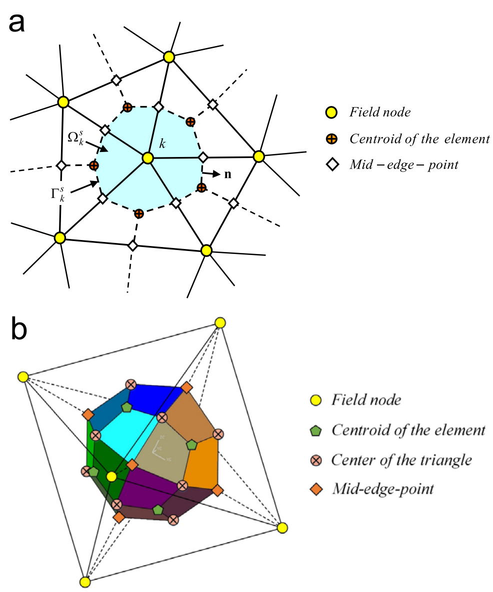

Fig. 1. The schematic of a node-based smoothing domain for node k. (a) 2D smoothing domain, and (b) 3D smoothing domain.

that are complementary to the FEM and can be applied easily to tetrahedral or triangular elements without any modification in the formulation and procedures [9]. NS-FEM won the favor recently for its prominent inherent properties [9]. Such as its insensitive to element distortion, and it is well immune from the volumetric locking and so on. And also, the computation time and computation efficiency of NS-FEM have been studied in previous works using bandwidth solver for linear elasto-statics [38]. Researchers found the computational efficiency of NS-FEM was three times lower than that of FEM-T3 in terms of displacement norm, 20 times higher in strain energy norm [39].

In smoothed FEM models, two types of instability, spatial stability and temporal stability, have been found [39], which is the major difficulty for node-based solutions. A spatially stable model always produces a unique and convergent solution for static problems when functions are bounded. However, this does not guarantee a stable solution for dynamic problems [40]. The "temporal instability" is defined as models that have spurious non-zero eigen modes. Such models are spatially stable, and will not have zero energy modes. However, when they are excited at (strictly non-zero) higher energy level, it can behave un-physically [39]. The NS-FEM-T3 has proven spatially stable [38], but found temporally instable. Even when a unconditionally stable time-integration scheme is used to solve transient dynamic problems [39], un-physical numerical responses can appear.

Beissel and Belytschko [41] proposed a stabilized nodal integration procedure by adding a residual of the equilibrium equation to the potential energy functional in an element-free Galerkin (EFG) framework. Researchers [39] further studied this solution into NS-FEM situation, and found that it could diminish spurious near-singular modes with a proper α value. However, its main defects are [8]: (1) for problems that do not contain unstable modes in their original solution, the addition of stabilization

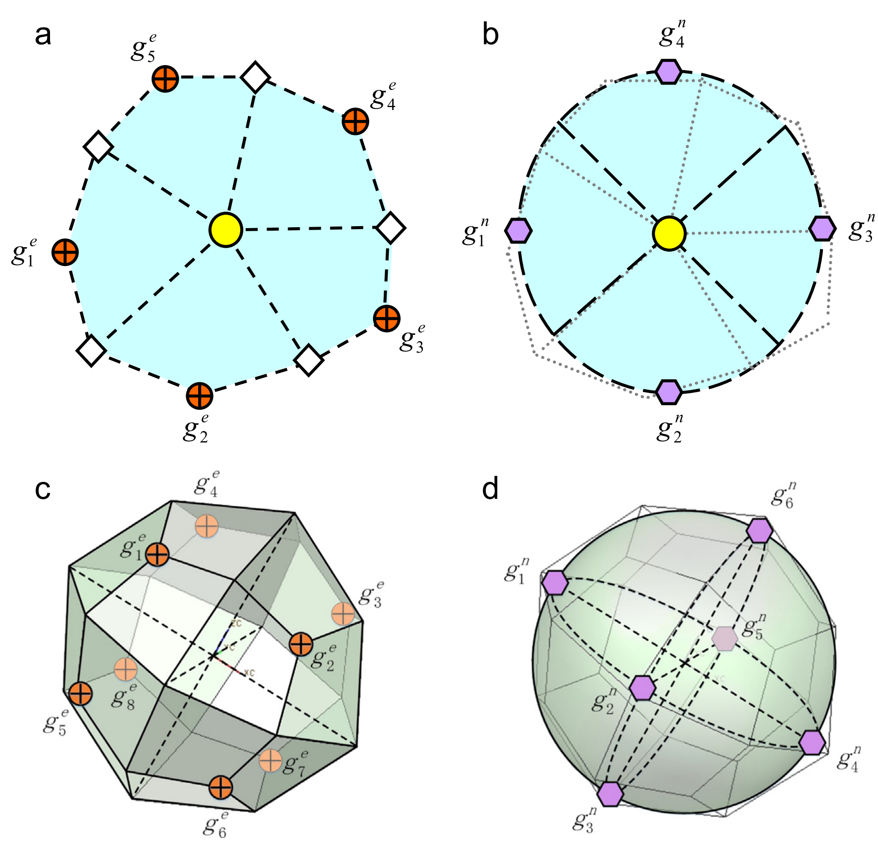

Fig. 2. The integration domain and integration points for FEM and SNIM-SG for 2D and 3D problems. (a) FEM integration for 2D, (b) SNIM-SG integration for 2D, (c) FEM integration for 3D, and (d) SNIM-SG integration for 3D.