Engineering Analysis with Boundary Elements 62 (2016) 112–122

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Efficient SPH simulation of time-domain acoustic wave propagation

Y.O. Zhang [a,b], T. Zhang [a,c,*], H. Ouyang [d], T.Y. Li [a,c]

[a] School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
[b] Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093, USA
[c] Hubei Key Laboratory of Naval Architecture and Ocean Engineering Hydrodynamics, Huazhong University of Science and Technology, Wuhan 430074, China
[d] School of Engineering, University of Liverpool, The Quadrangle, Liverpool L69 3GH, UK

ARTICLE INFO

Article history:
Received 12 January 2015
Received in revised form
3 August 2015
Accepted 22 September 2015
Available online 2 November 2015

Keywords:
Acoustic wave equations
SPH
Taguchi method
ANOVA
Time domain
Meshfree method

ABSTRACT

As a Lagrangian meshfree method, smoothed particle hydrodynamics (SPH) can eliminate much of the difficulty in solving acoustic problems in the time domain with deformable boundaries, complex topologies, or those that consist of multiphase systems. However, the optimal value of the computational parameters used in the SPH simulation of acoustics remains unknown. In this paper, acoustic wave equations in Lagrangian form are proposed and solved with the SPH method to compute the two-dimensional sound propagation model of an ideal gas in the time domain. We then assess how the numerical error is influenced by the time step, the smoothing length, and the particle spacing by investigating the interaction effects among the three parameters using Taguchi method with orthogonal array design (OAD) and analysis of variance (ANOVA). On the basis of this assessment, appropriate values for these computational parameters are discussed separately and validated with a two-dimensional computational aeroacoustic (CAA) model. The results demonstrate that the Courant number for the meshless SPH simulation of two-dimensional acoustic waves is proposed to be under 0.4, whereas the ratio of the smoothing length to the particle spacing is between 1.0 and 2.5.

1. Introduction

Numerical methods are widely used in sound wave propagation analysis, and computational acoustics are currently applied in industrial applications such as noise reduction of fans and manufactures, underwater detection, and room acoustics. Many classic numerical methods have been applied in spectral or temporal acoustic simulations; these methods include finite difference method (FDM) [1], finite element method (FEM) [2], boundary element method (BEM) [3], and other modified or coupled methods [4,5]. However, all of these mesh-based methods require a good-quality mesh generated step before computation, thereby increasing computational costs and human labor.

Meshfree methods [6] have recently attracted considerable interest for modeling acoustic waves with a set of arbitrarily distributed nodes, because these methods can avoid mesh generation and handle acoustic problems with complex geometric boundaries or with large ranges of density. Several meshfree methods have been introduced in acoustic simulation, such as method of fundamental solutions (MFS) [7], multiple-scale reproducing kernel particle method (RKPM) [8], meshless Galerkin least-square (MGLS) method [9], element-free Galerkin method (EFGM) [10], and a number of hybrid methods [11,12]. Most applications of these meshfree methods are focused on solving the Helmholtz equation, and they can provide very accurate results [13].

In view of its Lagrangian property, the SPH method not only possesses the many advantages of a meshfree method but is also suitable for solving problems with moving or deformable boundaries, multiphase systems, and object separation in the time domain, as demonstrated in recent reviews by Liu et al. [14], Liu and Liu [15], Springel [16], and Monaghan [17]. To date, this method has been successfully applied in many different fields [18–21]. By introducing this method to acoustic simulation, we can utilize its advantages for specific fields, such as combustion noise, bubble acoustics, and sound propagation in multiphase flows. However, the optimal values of computational parameters, like the time step, the smoothing length, and the particle spacing, are still unknown, because the SPH method was just applied to simulate sound waves in recent years.

The SPH method was first independently pioneered by Lucy [22], and Gingold and Monaghan [23] in 1977 to solve astrophysical problems. The method computes results using a set of particles possessing individual material properties. The first

* Corresponding author at: School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, Wuhan 430074, China. Tel.: +86 13995559242.
E-mail addresses: zhangyo1989@gmail.com (Y.O. Zhang), zhangt7666@mail.hust.edu.cn (T. Zhang), h.ouyang@liverpool.ac.uk (H. Ouyang), ltyz801@mail.hust.edu.cn (T.Y. Li).

http://dx.doi.org/10.1016/j.enganabound.2015.09.007
0955-7997/

attempt at simulating sound waves was realized by solving fluid dynamic equations in 2008 [24] and then a similar work was given in 2009 [25].

However, for various acoustic waves in engineering problems, acoustic variables such as the variation in pressure, density, and velocity are generally small. On the contrary, the values of pressure, density, and velocity exist on a much larger scale than any variation in these variables, as shown in Chapter 1 in [26]. Consequently, solving acoustic wave equations requires a lower computational burden compared to solving the fluid dynamic equations to simulate acoustic waves, and this approach had been widely used in modeling engineering problems [25–29]. Considering this fact, we solved the acoustic wave equations, and gave some one-dimensional tests, like the sound propagation and interference model [30,31], the sound reflection model [32]. The SPH simulation results show good agreement with theoretical solutions. But the computational parameter values for acoustic simulation have not been discussed in any research, and only one-dimensional model was given in literatures. Therefore, this paper focuses on discussing optimal values of three computational parameters, namely the time step, the smoothing length, and the particle spacing, for SPH simulation of two-dimensional acoustic waves based on the Taguchi method with considering the interactions among different parameters.

Taguchi method [33], also known as the orthogonal array design (OAD), is applied for the evaluation of different computational parameters in the present paper. This method uses a special design of orthogonal arrays to study the entire parameter space with only a small number of experiments [34], and the analysis is always given with the analysis of variance (ANOVA) techniques. As an optimization method, Taguchi method had been used in the parameter optimization of finite element model [35], the engineering design [36], the evaluation of combined effects of different parameters [37] etc. Experiments for analysis in this paper are numerical experiments modeling with the SPH method.

The present paper is organized as follows. In Section 2, the acoustic wave equations in Lagrangian form are given and solved using the standard SPH theory. In Section 3, a two-dimensional sound propagation model is tested to validate the SPH acoustic formulations, and the significance of different computational parameters and the interactions among them are analyzed using Taguchi method and ANOVA techniques. In Section 4, suitable values of these computational parameters with significant effects are discussed based on the Taguchi result. After that, a two-dimensional computational aeroacoustic (CAA) model is computed to verify the optimized parameters in Section 5, while Section 6 summarizes the results of this work.

2. SPH formulations of acoustic waves

2.1. Basic concepts of SPH

Functions in the SPH method are represented in a particle approximation form. Some basic concepts [38] are shown in this section.

A function $f(\boldsymbol{r})$ can be represented as

$$\langle f(\boldsymbol{r})\rangle = \int_{\Omega} f(\boldsymbol{r}')W(\boldsymbol{r}-\boldsymbol{r}', h)\mathrm{d}\boldsymbol{r}' \tag{1}$$

where f is a function of the vector $\boldsymbol{r}$, Ω is the volume of the integral, W is the smoothing kernel, and h is the smoothing length. The kernel approximation operator is marked by the angle bracket $\langle\rangle$.

The particle approximation for the function $f(\boldsymbol{r})$ at particle i can be written as

$$\langle f(\boldsymbol{r}_i)\rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{r}_j)\cdot W_{ij} \tag{2}$$

where r_i and r_j are positions of particles i and j, N is the number of particles in the computational domain, m_j is the mass of particle j, $W_{ij} = W(\boldsymbol{r}_{ij}, h)$, and $\boldsymbol{r}_{ij}$ is the distance vector from particle i to particle j.

Similarly, to substitute $f(\boldsymbol{r}_i)$ with $\nabla\cdot f(\boldsymbol{r}_i)$, the spatial derivative $\nabla\cdot f(\boldsymbol{r}_i)$ is obtained as

$$\langle \nabla\cdot f(\boldsymbol{r}_i)\rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{r}_j)\cdot \nabla_i W_{ij} \tag{3}$$

where $\nabla_i W_{ij} = \frac{\boldsymbol{r}_i-\boldsymbol{r}_j}{r_{ij}}\frac{\partial W_{ij}}{\partial r_{ij}}$.

The cubic spline function given by Monaghan and Lattanzio [39] is used as the smoothing kernel in the present paper.

2.2. Acoustic wave equations in Lagrangian form

In fluid dynamics, the fluid motion is defined by using the laws of continuity, momentum and energy. The equation of continuity in Lagrangian form, as shown in Chapter 4 of [38], is

$$\frac{D\rho^{\mathrm{L}}}{Dt} = -\rho^{\mathrm{L}}\nabla\cdot \boldsymbol{u}^{\mathrm{L}} \tag{4}$$

where ρ^{L} is the fluid density and $\boldsymbol{u}^{\mathrm{L}}$ is the flow velocity associated with a fluid particle at time t, superscript L stands for the Lagrangian variable associated with a fluid particle. The Lagrangian derivative [40] is defined by

$$\frac{D\rho^{\mathrm{L}}}{Dt} = \frac{\partial\rho^{\mathrm{L}}}{\partial t} + (\boldsymbol{u}^{\mathrm{L}}\cdot\nabla)\rho^{\mathrm{L}} \tag{5}$$

The simplest and most common acoustical problem occurs when body forces are not significant and the medium is characterized as inviscid and thermally nonconducting. In this case, the equation of momentum in Lagrangian form is

$$\frac{D\boldsymbol{u}^{\mathrm{L}}}{Dt} = -\frac{1}{\rho^{\mathrm{L}}}\nabla P^{\mathrm{L}} \tag{6}$$

where P^{L} is the instantaneous pressure of a fluid particle at time t.

Now suppose, on one hand, the medium is lossless and at rest, so an energy equation is unnecessary; on the other hand, a small departure from quiet conditions occurs, expressed in writing

$$\rho^{\mathrm{L}} = \rho_0^{\mathrm{L}} + \delta\rho^{\mathrm{L}}, \quad |\delta\rho^{\mathrm{L}}| < < \rho_0^{\mathrm{L}} \tag{7}$$

$$P^{\mathrm{L}} = p_0^{\mathrm{L}} + \delta p^{\mathrm{L}}, \quad |\delta p^{\mathrm{L}}| < < \rho_0^{\mathrm{L}} c_0^2 \tag{8}$$

$$\boldsymbol{u}^{\mathrm{L}} = 0 + \boldsymbol{u}^{\mathrm{L}}, \quad |\boldsymbol{u}^{\mathrm{L}}| < < c_0 \tag{9}$$

where ρ_0^{L} is the quiescent density which does not vary in time and space, $\delta\rho^{\mathrm{L}}$ is the change in density, p_0^{L} is the quiescent pressure, δp^{L} is the sound pressure, and all these variables are associated with a fluid particle; c_0 is the speed of sound.

In Eqs. (7)–(9), the inequalities at the right mean that $\delta\rho^{\mathrm{L}}$, δp^{L}, and $\boldsymbol{u}^{\mathrm{L}}$ are taken to be "small quantities of first order". Substitute these expressions into the continuity and the momentum equations (Eqs. (4) and (6)):

$$\frac{D(\rho_0^{\mathrm{L}} + \delta\rho^{\mathrm{L}})}{Dt} = -\rho^{\mathrm{L}}\nabla\cdot\boldsymbol{u}^{\mathrm{L}} \tag{10}$$

$$\frac{D\boldsymbol{u}^{\mathrm{L}}}{Dt} = -\frac{1}{\rho^{\mathrm{L}}}\nabla(p_0^{\mathrm{L}} + \delta p^{\mathrm{L}}) \tag{11}$$