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Boundary element method based vibration analysis of elastic bottom plates of fluid storage tanks resting on Pasternak foundation

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A R T I C L E I N F O

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A B S T R A C T

A higher-order boundary element procedure is presented for the free vibration analysis of flexible base plates of rigid fluid storage tanks resting on elastic foundation. The main principles of the procedure are replacing the biharmonic operator of the thin plate vibration problem by two successive harmonic operators, and representing the forcing terms (plate inertia, fluid loading and foundation influence) by applying the dual reciprocity boundary element formulation. The fluid effect on the plate dynamics is incorporated into the analysis by invoking another boundary element solution, which expresses the fluid pressure over the plate surface in terms of plate deflection. The performance of the method is thoroughly investigated and the nature of dynamic plate–foundation–fluid interaction is studied from several perspectives. The method provides excellent predictions, within the limits of Kirchhoff plate, potential flow, and Pasternak foundation models.

1. Introduction

Plates are essential structural elements that are widely adopted in various forms and configurations for different areas of engineering practice. Many of the related problems include some form of interaction with another medium, like soil, fluid, or both, which may substantially change the mechanical performance of the plate. Different mediums generally introduce new considerations to the problem, in turn may require adaptation of different solution techniques, due to the convenience they introduce. A common solution method for all interacting fields, however, can simplify and even improve the representation of the mutual coupling, and so the analysis of the overall system. A representative and widely encountered case that embodies such an interaction problem, requiring accurately assessment of the effects imposed by the external mediums on the plate, is the dynamic analysis of a base plate of a fluid storage tank, which also lies on an elastic foundation. Unlike the analytical or approximate analysis techniques, a numerical method capable of mimicking the behavior of all components and their interrelation can be readily applied to any form in this case and also be adaptive to complexities that the system may possess.

Contact between plates and fluid medium is common in structural engineering branches, such as civil, offshore, or aerospace engineering. Several analysis methods suggested by the researchers target different plate and fluid models, plate geometries, fluid spaces, structural configurations, free-surface effects, etc. Among the recent works on the dynamics of plate-fluid systems with additional rigid or elastic surfaces bounding the fluid domain partially or completely, as in the case of the bottom plate of a rigid container, the Rayleigh–Ritz method is the most preferred solution approach. Kim and Lee [1] studied an annular plate partially covering the free-surface of an otherwise rigid tank and presented both the plate bulging and fluid sloshing modes. Askari et al. [2] investigated the effect of a submerged rigid internal thin-walled cylinder on the free vibration of the bottom plate of a fluid-filled rigid container and also assessed the performance the cylinder for suppressing the free-surface waves; they applied the Galerkin method for tuning the two fluid velocity potential distributions that are described in separate parts of the non-convex fluid domain. In a similar study, the vibration of circular plates immersed in a rigid container is studied by Askari et al. [3], to examine the effect of a submerged plate as a sloshing suppression device; the fluid motion is described by means of the least square and Galerkin methods here. Jeong and Kang [4] obtained dynamic characteristics of multiple rectangular plates that are submerged in a liquid confined by rigid surfaces, representative of fuel plates for heat exchange in coolant flow. Hasheminejad and Tafani [5] studied the hydroelastic vibrations of the elliptic bottom plate of a cylindrical container, with additional interest at the coupling between fluid sloshing and plate modes; their analytical solution is based on the modal expansion of the plate deflection in terms of radial and angular Mathieu functions. The primary

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advantages of the above-mentioned approximate or analytical studies are direct access to specific modes (e.g., axisymmetric modes), which enables more systematic qualitative analysis, and generally shorter simulation times. Numerical approaches, on the other side, are more flexible and can be applied to a variety of conditions. They appear generally in the form of finite element method (FEM) or FEM with the boundary element method (BEM), where the FEM is used for the plate, and BEM for the fluid with its versatility to deal with complex or infinite domains. Ergin and Uğurlu [6] studied partially or completely filled fluid storage tanks of different arrangements – rigid tank with flexible bottom, flexible wall with rigid bottom, and complete flexible structure – through a reduced order FEM/BEM model. Kerboua et al. [7] presented a plate finite element using classical shell theory and analyzed free vibration of plates interacting with fluid domain, which is also modeled by finite elements, through different configurations.

The plate supported by an elastic foundation is a well-accepted approximation for practical soil–structure interaction problems. Among the models that are proposed to represent the behavior of soil continuum [8], the Pasternak foundation assumes the contact pressure at any point of the structure is proportional to the displacement of the soil at that point, like the Winkler foundation, but also considers the effect of the neighborhood through permitting shear interaction with other points. This two-parameter model is represented by a distribution of springs and a shear layer relating the springs. Several studies have been conducted on the free vibration of plates resting on Pasternak foundation. Omurtag and Kadıoğlu [9] developed a finite element solution for orthotropic thin plates by using the Gateaux differential method. Zhou et al. [10] adopted the three-dimensional elasticity theory and Rayleigh–Ritz method to study thick rectangular plates. Dehghan and Baradaran [11] combined the finite element and differential quadrature methods, for in-plane domain and along the thickness discretization, respectively, for vibration and buckling analysis of thick rectangular plates. In recent years, research is focused on solutions with broader coverage. Akhavan et al. [12] presented a closed-form solution for moderately thick rectangular plates subjected to uniaxial in-plane loading. Motaghian et al. [13] applied the separation of variables technique on thin rectangular plates supported partially by the Pasternak foundation. Jahromi et al. [14] studied partial interaction between moderately thick plates and Pasternak foundation by adopting the Mindlin theory and applying the generalized differential quadrature method to set the eigenvalue equations. Bahmyari and Khedmati [15] used the element free Galerkin method to investigate point supported non-homogeneous moderately thick plates lying on elastic foundation.

The interest on combined dynamic analysis of plate–foundation–fluid systems is relatively new. Uğurlu et al. [16] studied the free vibration of thin rectangular plates in contact with Pasternak foundation on one side and partially interacting with unconfined fluid domain on the other side by relating a mixed type finite element formulation for the plate–foundation system and fluid boundary element analysis. Hosseini Hashemi et al. [17] addressed the same problem with a Rayleigh–Ritz solution based on Timoshenko beam functions, by adopting the Mindlin plate and confining the fluid domain along its length and width through rigid surfaces. The vibration and buckling of rectangular bottom plate of a rigid fluid storage container resting on a Pasternak foundation and subjected to in-plane loads are studied by Hosseini-Hashemi et al. [18] with a similar approach. Kutlu et al. [19] improved the physical model in [16] by adopting the Mindlin plate and proposing a three parameter orthotropic Pasternak foundation.

Dynamic analysis of plates by the BEM covers an extensive list of solutions and applications [20,21]. The classical BEM relies on free-space fundamental solutions to transfer all terms of the related differential equation to the boundary. In case of plate–foundation interaction, the fundamental solutions are provided by Wen et al. [22] for thin plates on Winkler foundation, and by Wen [23] for moderately thick plates on Pasternak foundation. Despite its very high accuracy, the classical BEM suffers from complicated numerical treatment due to the form of the dynamic fundamental solutions, leading to development of alternative approaches. One effective strategy is replacing the dynamic fundamental solution, either with the static fundamental solution [24] or equivalent fictitious sources [25], both of which enforce the integral equation to involve extra domain expressions. Another approach is using a simpler fundamental solution and transferring/eliminating the remaining forcing terms by introducing a set of successive higher order fundamental solutions [26] or approximating the forcing terms as a combination of interpolation functions [27], so that preserving the boundary character of the solution. These simpler to apply approaches generally allow considering additional complexities, like internal supports, elastic foundations, nonlinearities, etc., in a relatively effortless way. The use of higher order fundamental solutions, known as multiple reciprocity method (MRM), has the additional advantage of providing boundary only discretizations, but with a computational cost and limited applicability. The recursive composite MRM [28] avoids the drawbacks by introducing higher order composite differential operators when eliminating the forcing term.

The present study proposes a boundary element method for the free vibration analysis of flexible bottom plates of rigid fluid storage tanks resting on Pasternak foundation. The study can be taken as an extension of a previous work [27], which adopted the idea introduced by Paris and de Leon [29]. The main principles of the procedure are replacing the biharmonic operator of the thin plate vibration problem by two successive harmonic operators, so allowing for boundary integral equations with simpler properties, and representing the plate inertia, fluid loading and foundation influence, the forcing terms in the original differential equation, by applying the dual reciprocity BEM formulation. The contained fluid effect on the plate dynamics is incorporated into the analysis by invoking another boundary element solution, which expresses the fluid pressure over the plate surface in terms of plate deflection. The performance of the method is investigated through several cases and dynamic behavior of bottom plates is studied extensively considering different perspectives.

2. Boundary element representation of the fluid-structure-foundation system

2.1. Dynamics of the coupled system

The equation of motion for the lateral vibrations of a homogeneous, isotropic, linear elastic thin plate resting on a Pasternak type foundation is given by [20]

$$D\nabla^4 W + k_1 W - k_2 \nabla^2 W + \rho h W_{,tt} = P. \tag{1}$$

Here $W(x, y; t)$ is the lateral displacement of the plate middle surface, with x, y axes forming the plate plane and t is time, h is the thickness, ρ is the mass density, $D = E\,h^3/12\,(1-\nu^2)$ is the flexural stiffness, P is the lateral dynamic load per unit area of the plate, and E, ν represent the Young modulus and Poisson ratio, respectively. Foundation related k_1 and k_2 are the sub-grade reaction parameter and shear modulus, respectively. If P is harmonic in time, that is $P(x,y;t) = p(x,y)e^{i\omega t}$ with ω denoting the angular frequency and p the amplitude, W also harmonically dependent on time, and Eq. (1) can be written as

$$D\nabla^4 w + k_1 w - k_2 \nabla^2 w - \rho h \omega^2 w = p. \tag{2}$$

Here $w(x, y)$ is the vibration amplitude. Considering the free vibrations of the bottom plate of an otherwise rigid fluid storage