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Method of approximate particular solutions for constant- and variable-order fractional diffusion models

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ABSTRACT

The method of approximate particular solutions (MAPS) is an alternative radial basis function (RBF) meshless method, which is defined in terms of a linear combination of the particular solutions of the inhomogeneous governing equations with traditional RBFs as the source term. In this paper, we apply the MAPS to both constant- and variable-order time fractional diffusion models. In the discretization formulation, a finite difference scheme and the MAPS are used respectively to discretize time fractional derivative and spatial derivative terms. Numerical investigation examples show the present meshless scheme has highly accuracy and computationally efficiency for various fractional diffusion models.

1. Introduction

In recent decades, anomalous diffusion phenomena are extensively observed in a wide range of engineering and physics fields [1–4], such as contaminant transport, seepage, magnetic plasma, dissipation and turbulence. To describe anomalous diffusion phenomena, constant-order fractional diffusion equations are considered as recent alternative models and have received fantastic success [5–7]. However, various recent experimental results [8,9] show that constant-order fractional diffusion equations cannot fully capture some more complicated diffusion processes, whose diffusion behaviors depend on the time evolution, spatial variation or even concentration variation. To deal with these issues, variable-order fractional diffusion equations [10,11] have been introduced, in which the variable-order time fractional operator can be time-dependent, spatial-dependent, and/or concentration-dependent.

Nowadays, finite difference methods (FDMs) are popular and dominant numerical techniques for temporal and spatial discretization of constant-order [12–16] and variable-order [17–20] fractional diffusion equations. Their convergence, accuracy, and stability have extensively been discussed in the literatures [21–24].

For the numerical simulations of constant-order fractional diffusion equations, with traditional FDMs for temporal discretization, several numerical methods have been introduced to spatial discretization of fractional derivative equations, such as the Fourier method [25], spectral method [26], finite element method [27–29], boundary element method [30], and radial basis function meshless collocation method [31–33]. In comparison with traditional FDMs for spatial discretization, these methods can reduce, to a certain extent, computing costs for large computational domain problems. In this work, we shall extend the idea to mitigate the computing costs in the numerical simulation of variable-order fractional diffusion equations.

We will focus on constant- and variable-order time fractional diffusion equations, which only have fractional derivative in time and integer differential operator in space. We employ a finite difference method for temporal discretization and introduce an alternative radial basis function (RBF) meshless method, the method of approximate particular solutions (MAPS) [34–37], for spatial discretization. Chen et al. [38] first proposed the method of approximate particular solutions (MAPS) to solving partial differential equations. Then the MAPS has been successfully applied to various physical and engineering problems, such as anisotropic problems [39], nonlinear Poisson problems [40], wave problems [41], elasticity problems [42], Stokes flow problems [43], and convection-diffusion problems [44]. In comparison with the famous RBF method, also known as the Kansa method, the MAPS uses a newly derived RBF as interpolation basis function, which include some information from the considered governing equation operator. And some numerical experiments [45–47] demonstrate that the MAPS outperforms the Kansa method in terms of both the stability and accuracy, particularly in the evaluation of partial derivatives.

This paper first applies the method of approximate particular solutions (MAPS), to 2D constant- and variable-order fractional diffusion problems. A brief outline of the paper is as follows. Section 2

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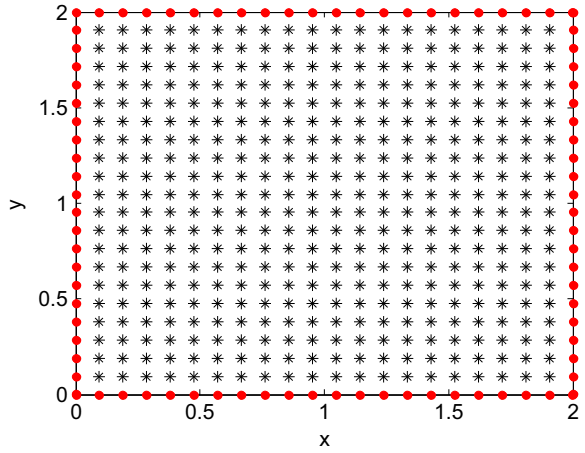

Fig. 1. Schematic configuration of uniform node distribution on a square domain (boundary nodes 'o' and inner nodes '*').

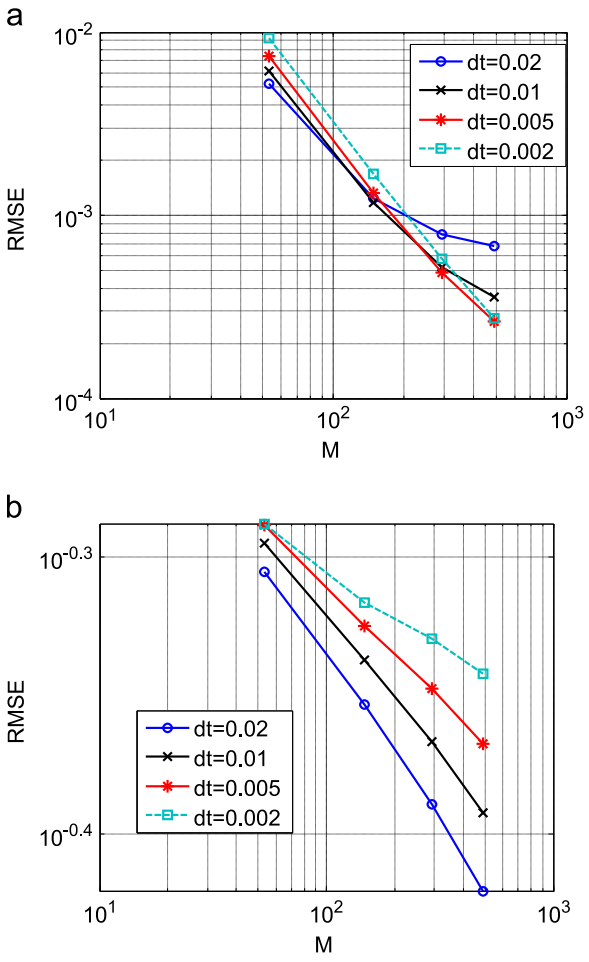

Fig. 2. Convergence rate (RMSE) of the present method with the derived RBF formulation (11(a)) by using different time steps (dt=0.02, 0.01, 0.005, 0.002) in Example 1. (a) Full Dirichlet boundary conditions and (b) mixed boundary conditions.

describes the present computational formulations for fractional diffusion equations. In Section 3, the efficiency and accuracy of the present approach are examined with some benchmark examples. Finally, Section 4 concludes this paper with some remarks.

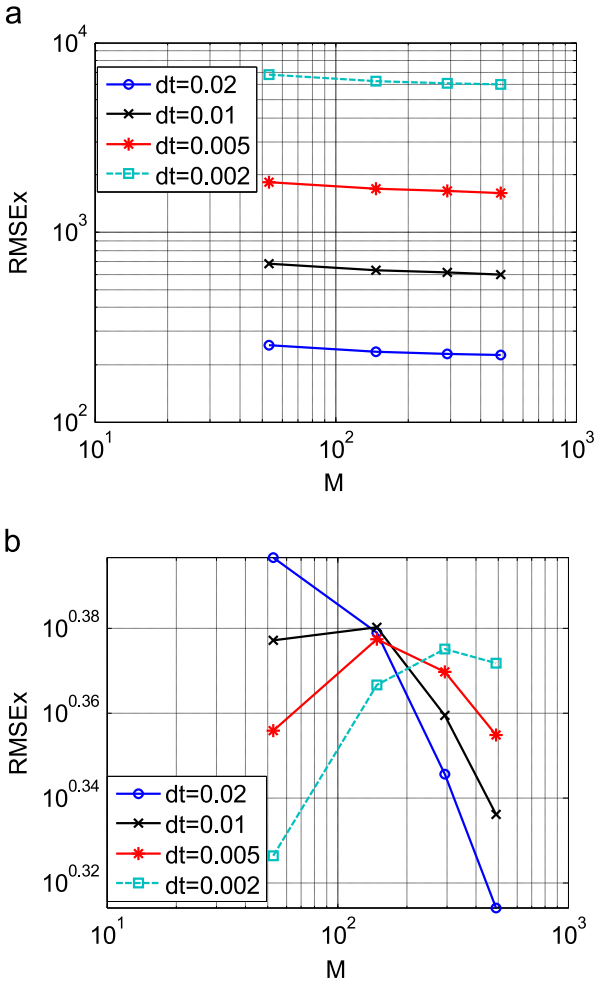

Fig. 3. Convergence rate (RMSEx) of the present method with the derived RBF formulation (11(a)) by using different time steps (dt=0.02, 0.01, 0.005, 0.002) in Example 1. (a) Full Dirichlet boundary conditions and (b) mixed boundary conditions.

2. Methodology

2.1. Time fractional diffusion model

Without loss of generality, we consider the following variable-order time fractional diffusion equations in a bounded domain Ω with piecewise smooth boundary $\partial\Omega=\Gamma_D+\Gamma_N$ ($\Gamma_D\cap\Gamma_N=\varnothing$)

$$\frac{\partial^{\alpha(t)}u(\mathbf{x},t)}{\partial t^{\alpha(t)}}=\left(D\Delta+\vec{v}\cdot\nabla-\lambda\right)u(\mathbf{x},t)+Q(\mathbf{x},t),$$
$$0<\alpha(t)<1,\quad \mathbf{x}\in\Omega,\quad t\in(0,T), \tag{1}$$

with boundary conditions

$$u(\mathbf{x},t)=g_1(\mathbf{x},t),\quad \mathbf{x}\in\Gamma_D,\quad t\in(0,T), \tag{2a}$$

$$\frac{\partial u(\mathbf{x},t)}{\partial n}=g_2(\mathbf{x},t),\quad \mathbf{x}\in\Gamma_N,\quad t\in(0,T), \tag{2b}$$

and initial condition

$$u(\mathbf{x},0)=u_0(\mathbf{x}),\quad \mathbf{x}\in\Omega, \tag{3}$$

where $Q(\mathbf{x},t)$, $g_1(\mathbf{x},t)$, $g_2(\mathbf{x},t)$ and $u_0(\mathbf{x})$ are known functions; D the diffusion coefficient, λ the reaction coefficient, $\vec{v}$ the velocity vector, n the unit outward normal, T the total time to be considered,