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The MFS as a basis for the PIM or the HAM – comparison of numerical methods

Anita Uscilowska

Institute of Material Technology, Poznan University of Technology, Piotrowo Street 3, 60-964 Poznan, Poland

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ABSTRACT

The aim of this paper is to present implementation of the Method of Fundamental Solutions. Using the MFS the fundamental solution of the operators appearing in the governing equations should be known. For many engineering problems the governing equations are linear with unknown fundamental solutions or nonlinear. The purpose of this paper is implementation of the Picard Iterations Method or Homotopy Analysis Method in such case. Both methods are supported by the MFS. Some engineering problems described by linear equation with unknown fundamental solution and system of nonlinear equations are considered. The numerical experiment, solving these engineering problems, is performed using both methods. The correctness of the results obtained by both methods is checked. The conditions of the convergence of both methods are described.

1. Introduction

The Method of Fundamental Solutions (the MFS) is a numerical method for solving boundary value problems. This method was proposed in [26] and numerically implemented for the first time in [35]. The requirement for the applicability of the MFS is that the governing equation of the problem in question should be homogeneous and its 'fundamental solution' should to be known. In the literature there are many papers presenting the solution of engineering problems obtained by the MFS. However, the governing equations in these papers are of Laplace, Helmholtz, modified Helmholtz, biharmonic, Navier type. Comprehensive reviews of the MFS for various applications can be found in [9,12]. Many engineering Boundary Value Problems (BVPs) are described by partial differential equations with other differential operators than the ones mentioned above. Moreover, more advanced description of many engineering problems uses nonlinear partial differential equations. Many problems of physics, mechanics are nonlinear, as the authors of [29] showed.

In mechanics of deformable solids and in fluid mechanics few sources of nonlinearity are indicated. The material or physical nonlinearity is caused by specific nonlinear structural properties. The temperature field in the medium with a thermal conductivity coefficient dependent on temperature is one of the examples of material nonlinearity. Another kind of material nonlinearity appears if material characteristic depends (i.e. Poisson ratio, Young modulus) on geometrical coordinates. The geometrical characteristics (as for example width of a plate dependent on geometrical coordinates) may introduce nonlinearity of the considered problem. Nonlinearity may result also from the nonlinearity of internal sources. One of the common examples of a such nonlinearity is the internal source in chemical reactions. Its density changes with the density of the reacting substance, temperature etc. Also the pressure field in a porous medium is described by the equation in which element corresponding to the internal sources is pressure dependent. So, it may lead to a nonlinearity.

The relation between the displacement and the deformation causes a geometrical nonlinearity, which appears in problems with variable boundaries. Therefore, nonlinear boundary conditions are sources of nonlinearity in mechanical problems, too.

It is worth to mention that problems with a geometrical nonlinearity or problems with nonlinear boundary conditions do not require any modification in implementing the MFS. Taking into account nonlinear boundary conditions leads to a system of nonlinear algebraic equations. So, a technique for solving a system of algebraic nonlinear equations is used, instead of a procedure for solving a system of algebraic linear equations.

Nonlinearities caused by nonlinear material characteristics or nonlinear internal sources impact on governing differential equations and lead to its nonlinearity. Solving such problems requires a modification of the method, which is used for solving linear problems.

The issue of solving nonlinear problems is rather complicated, so it is very widely discussed in the literature. The results however, are not satisfactory and the question of solving nonlinear problems is still open.

In the literature there are some proposals for solving such problems and it is worth mentioning some of them.

The authors in [23] proposed the quasi-linear Boundary Elements Method (BEM) for solving nonlinear problems, which are

E-mail address: anita.uscilowska@put.poznan.pl

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described by equations which contain the Laplace operator. The inhomogeneous part of the equation, which includes the nonlinear part of the equation operator, is approximated by Taylor series and the linear elements of this series are taken into account. In this way the linear BVP is obtained and is solved by the classical BEM. In paper [63] the inhomogeneous part of the equation, which includes the nonlinear part of the equation operator, and the general solution is approximated by monomials. The linear BVP, obtained in this way, is solved using the BEM. This method may be used for a very limited set of problems (due to the approximation by the monomials). Another paper, [66], proposes the use of the Local Boundary Integral Equation Method. Both linear and nonlinear problems of second order are solved in [66].

The modification of the MFS for solving quasi-linear Dirichlet-type problems is proposed in [3]. The authors of [3] calculate the approximation of the Green function for the quasi-linear equation and apply a procedure similar to the MFS.

The problem of heat transfer in a medium with a heat conductivity coefficient dependent on temperature is quite widely discussed in the literature. The authors of paper [24] implement the Kirchhoff transformation for function describing material characteristics. The pseudolinear problems obtained are solved by the BEM. In paper [14] the results of implementing several numerical methods for the problem described above are presented. In these methods the derivative with respect to time variable has been approximated by different ways (implicit, explicit Method of Finite Differences, BEM and Method of Boundary Integral Equations). To solve the BVP the BEM was applied. The authors of [1] implemented the Kirchoff transformation. The obtained linear BVP is solved by Trefftz method using the T-*complete* functions. The disadvantage of such an approach is need to calculate the integrals.

The nonlinear problem of calculating a minimal surface is solved in paper [60]. The procedure implemented is very complex: it is based on the hybrid Trefftz finite element method and the approximation by the Radial Basis Functions (RBF). To oust the nonlinearity of the governing equation the Method of Analog Equation is used. The generalised form of the Multireciprocity Method BEM (MR-BEM) applied to nonlinear potential problems is proposed in paper [38]. For dealing with the nonlinearity, the Kirchhoff transformation is used. The problem of heat transfer in a medium with variable material parameters is also solved by some methods presented in [47], too. The Picard Iterations Method (PIM) and Newton method have been tested for solving the nonlinear problem. It is possible to implement these methods for problems described by the equation, which includes linear part of differential operator. At each iteration step the BVP is obtained, which consists of a transient equation. These BVPs are solved by the BEM using a time-dependent fundamental solution. In the paper [5] the authors transformed the equations and boundary conditions of the Initial-Boundary Value Problem of nonlinear heat transfer to a form for genetic algorithm implementation. The results obtained show that the proposed method gives very good results, but is extremely time-consuming. Another proposal solving this problem is included in [34]. Derivatives with respect to time are approximated by finite differences. So, at each time step a BVP consisting of the Laplace operator is obtained. To solve such the BVP the authors proposed to implement the Homotopy Analysis Method (the HAM). They proposed and described the HAM in papers [30,31]. This method is more general than an iterative scheme (Picard, Newton). It does not have limitations in that the governing equation should consist of a linear part. The implementation of the HAM supported by the MFS for some engineering problems is presented in [58,54].

In present paper the HAM-MFS applications are extended to another complicated engineering problems.

In [64] the solution of heat transfer in polymers is presented. The authors implemented the BEM with the PIM.

In [65] the authors solved problem described by nonlinear transient diffusion equation. The nonlinear part of the equation is linearised and the BVP obtained is solved using the Laplace transformation. The same problem is solved in [45] by applying the Kirchhoff transformation. The authors of [62] solved a transient diffusion problem using the direct MFS with time-dependent fundamental solutions.

The other engineering problems solved by the BEM supported by iterations are elastic problems in a medium with nonlinear geometrical characteristics. The solution of these problems is presented in [40].

The other group of nonlinear engineering problems widely discussed in the literature is the deflection of a plate (Kirchhoff plate, von Karaman plate). The Kirchhoff plate was considered in [10]. The authors implemented an iterative scheme and the BEM. The authors of [28] solved the problem of the Kirchhoff plate using the RBFs presented in [20,21]. In [39] a governing equation is rewritten as two Poisson equations and the BEM is then applied.

The problem of large deflection of a thin plate is considered in [61]. The authors transformed the two governing nonlinear equations to biharmonic ones. The system of equations with boundary conditions is solved by iterations supported by the BEM. The same problem is considered in [19,48,49]. The solution is obtained by the BEM and the Method of Surface Elements. The authors in [37] solve the same problem using Kansa's Method, and the authors in [57] use the MFS. A Collocation Method is applied for solving this problem by the authors in [8].

Another nonlinear problem is the dynamic analysis of porous medium. The authors in [46] solve this problem by the BEM implemented for transient problems. The flow problem in a porous medium is considered in [7] and it is solved by the BEM. In paper [15] Darcy's equation is linearised by some assumption related to the region considered. The obtained BVPs for the Laplace equation are solved with the Finite Element Method. For the problem of laminar flow and convection in a porous medium the authors in [18] use the Finite Difference Method. Isothermic flow in a porous medium is considered in [59,56]. Here, the MFS supported by the PIM and the FDM are used. The problem of natural convection is considered in [44] and solved by iterations with DRM. The authors of [41] solve the same problem by the FDM.

There are many more engineering problems considered in the literature. In this section, only a few examples of mechanical problems have been presented. The numerical approach for solving these problems have been discussed.

A very large number of papers considering numerical algorithms and their application to engineering problems show that solving such problems is very important. The results obtained indicate the improvement of the proposed methods is necessary.

2. The numerical approach to nonlinear problems

In this paper the implementation the MFS will be presented for a certain group of engineering problems. As mentioned in the beginning the crucial information in application of the MFS is knowing the fundamental solution of governing equation of the problem considered. In [27] the general theory of fundamental solutions of widely used operators is presented. The authors in [27] proposed solutions for many problems of different scientific disciplines in form of the fundamental solutions.

The implementation of the MFS for BVPs with equations with unknown fundamental solutions or nonlinear equations cannot be direct, but should be combined with other methods. The purpose of this paper is to show the application of the PIM and the HAM supported by the MFS and the comparison of the numerical